Intermediate Algebra
Skill Builder # PF – 12
Factoring by Grouping II

Polynomials involving 4 or more terms may be factored by grouping and a combination of the other techniques.

Examples

1. $x^5 - 9x^3 + x^2 - 9$

Solution:

$$\begin{aligned} x^5 - 9x^3 + x^2 - 9 &= \left(x^5 - 9x^3\right) + \left(x^2 - 9\right) \\ &= x^3\left(x^2 - 9\right) + \left(x^2 - 9\right) \\ &= \left(x^2 - 9\right)\left(x^3 + 1\right) \\ &= (x-3)(x+3)(x+1)\left(x^2 - x + 1\right) \end{aligned}$$

2. $y^5 - 3y^4 + 2y^3 - 8y^2 + 24y - 16$

Solution:

$$\begin{aligned} & y^5 - 3y^4 + 2y^3 - 8y^2 + 24y - 16 \\ &= \left(y^5 - 3y^4 + 2y^3\right) + \left(-8y^2 + 24y - 16\right) \\ &= y^3\left(y^2 - 3y + 2\right) - 8\left(y^2 - 3y + 2\right) \\ &= \left(y^2 - 3y + 2\right)\left(y^3 - 8\right) \\ &= (y-1)(y-2)(y-2)\left(y^2 + 2y + 4\right) \end{aligned}$$

3. $a^2 - 2a + 1 - b^2$

Solution:

$$\begin{aligned} & a^2 - 2a + 1 - b^2 \\ &= \left(a^2 - 2a + 1\right) - b^2 \\ &= (a-1)^2 - b^2 \\ &= (a - 1 - b)(a - 1 + b) \end{aligned}$$

4. $x^2 + 2xy + y^2 - a^2 - 4a - 4$

Solution:

$$\begin{aligned} & x^2 + 2xy + y^2 - a^2 - 4a - 4 \\ &= \left(x^2 + 2xy + y^2\right) - \left(a^2 + 4a + 4\right) \\ &= (x+y)^2 - (a+2)^2 \\ &= (x + y - a - 2)(x + y + a + 2) \end{aligned}$$

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Factor.

1. $x^5 - 4x^3 + x^2 - 4$

2. $x^5 - x^3 - 8x^2 + 8$

3. $a^2x^2 - 4a^2 - 9x^2 + 36$

4. $x^2y^2 - 16y^2 - x^2 + 16$

5. $x^5 + x^4 - 6x^3 - x^2 - x + 6$

6. $6y^5 + y^4 - 2y^3 + 48y^2 + 8y - 16$

7. $x^2 + 8x + 16 - y^2$

8. $9y^2 - a^2 + 4a - 4$

9. $x^2 - 2x + 1 - y^2 - 2yz - z^2$

10. $4a^2 + 12ab + 9b^2 - 9x^2 - 12xy - 4y^2$

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Answers

1. $(x-2)(x+2)(x+1)\left(x^2-x+1\right)$
2. $(x-2)\left(x^2+2x+4\right)(x-1)(x+1)$
3. $(a-3)(a+3)(x-2)(x+2)$
4. $(x-4)(x+4)(y-1)(y+1)$
5. $(x-1)\left(x^2+x+1\right)(x+3)(x-2)$
6. $(3y+2)(2y-1)(y+2)\left(y^2-2y+4\right)$
7. $(x+4-y)(x+4+y)$
8. $(3y-a+2)(3y+a-2)$
9. $(x-1-y-z)(x-1+y+z)$
10. $(2a+3b-3x-2y)(2a+3b+3x+2y)$

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