Intermediate Algebra
Skill Builder # PF – 14
Factoring by Substitution

We can use the ***substitution method*** to reduce a polynomial to a form we can easily recognize and factor.

Examples

1. $36x^4y^8 - 49z^{12}w^6$

Solution:

Note that each term is a perfect square:

$$36x^4y^8 - 49z^{12}w^6 = \left(6x^2y^4\right)^2 - \left(7z^6w^3\right)^2$$

If we let $a = 6x^2y^4$ and $b = 7z^6w^3$, we then get

$$\begin{aligned} 36x^4y^8 - 49z^{12}w^6 &= \left(6x^2y^4\right)^2 - \left(7z^6w^3\right)^2 \\ &= a^2 - b^2 \\ &= (a-b)(a+b) \\ &= \left(6x^2y^4 - 7z^6w^3\right)\left(6x^2y^4 + 7z^6w^3\right) \end{aligned}$$

2. $8a^6b^3 + 27c^{24}$

Solution:

Note that each term is a perfect cube:

$$8a^6b^3 + 27c^{24} = \left(2a^2b\right)^3 + \left(3c^8\right)^3$$

If we let $x = 2a^2b$ and $y = 3c^8$, we then get

$$\begin{aligned} 8a^6b^3 + 27c^{24} &= \left(2a^2b\right)^3 + \left(3c^8\right)^3 \\ &= x^3 + y^3 \\ &= (x+y)\left(x^2 - xy + y^2\right) \\ &= \left(2a^2b + 3c^8\right)\left(4a^4b^2 - 6a^2bc^8 + 9c^{16}\right) \end{aligned}$$

3. $x^4 - 6x^2 - 27$

Solution:

If we let $y = x^2$, we get

$$\begin{aligned} x^4 - 6x^2 - 27 &= y^2 - 6y - 27 \\ &= (y-9)(y+3) \\ &= \left(x^2 - 9\right)\left(x^2 + 3\right) \\ &= (x-3)(x+3)\left(x^2 + 3\right) \end{aligned}$$

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Factor.

1. $16a^4 - 81b^8$

2. $\dfrac{9n^2}{m^{12}} - \dfrac{49w^4}{p^6}$

3. $64x^3y^6 + 27z^9$

4. $\dfrac{1}{8}a^9b^3 - \dfrac{1}{27}c^{15}$

5. $y^4 - 16y^2 + 63$

6. $n^6 + 5n^3 - 6$

7. $6x^4 + 31x^2 + 35$

8. $15a^6 + 31a^3 - 24$

9. $8x^{-2} + 6x^{-1} - 27$

10. $20y^{-2} - y^{-1} - 30$

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Answers

1. $\left(2a-3b^2\right)\left(2a+3b^2\right)\left(4a^2+9b^4\right)$
2. $\left(\frac{3n}{m^6}-\frac{7w^2}{p^3}\right)\left(\frac{3n}{m^6}+\frac{7w^2}{p^3}\right)$
3. $\left(4xy^2+3z^3\right)\left(16x^2y^4-12xy^2z^3+9z^6\right)$
4. $\left(\frac{1}{2}a^3b-\frac{1}{3}c^5\right)\left(\frac{1}{4}a^6b^2+\frac{1}{6}a^3bc^5+\frac{1}{9}c^{10}\right)$
5. $\left(y^2-7\right)\left(y-3\right)\left(y+3\right)$
6. $\left(n-1\right)\left(n^3+6\right)\left(n^2+n+1\right)$
7. $\left(2x^2+7\right)\left(3x^2+5\right)$
8. $\left(5a^3-3\right)\left(3a^3+8\right)$
9. $\left(2x^{-1}-3\right)\left(4x^{-1}+9\right)$
10. $\left(5y^{-1}+6\right)\left(4y^{-1}-5\right)$

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