Mini-Lecture 1.1
Study Skill Tips for Success in Mathematics

Learning Objectives:

1. Get ready for this course.
2. Understand some general tips for success.
3. Know how to use this text.
4. Know how to use text resources.
5. Get help as soon as you need it.
6. Learn how to prepare for and take an exam.
7. Develop good time management.

Examples:

1. Getting ready for this course.
   a) Positive attitude   b) Allow adequate time for class arrival
   c) Bring all required materials

2. General tips for success.
   d) Find a contact person   e) Choose to attend all classes
   f) Do your homework   g) Check your work
   h) Learn from mistakes   i) Seek help when needed
   j) Stay organized   k) Read textbook before class
   l) Ask questions   m) Hand in all assignments on time

3. Understand how to use the text.
   a) Practice Problems – a problem the student can try to match each example in each section.
   b) Chapter Test Prep Video CD – (CD inside book) each Chapter test exercise worked out by the author.
   c) Lecture Video CDs – (not included with book) the author explains each section and works specified example problems within each section
   d) Review the meaning of icons used in text.
   e) At beginning of each section, a list of icons shows availability of support materials.
   f) Integrated Reviews – found in the middle of each chapter for students to practice the concepts previous learned in a chapter
   g) Each chapter ends with Chapter Highlights, Reviews, and Practice Tests.

4. Learn to take exams.
   a) Review previous homework assignments, class notes, quizzes, etc.
   b) Read Chapter Highlights to review concepts and definitions.
   c) Practice working out exercises in the end-of-the-chapter Review and Test.
   d) When taking a test, read directions and problems carefully.
   e) Pace yourself. Use all available time. Check your work and answers.

5. Good time management.
   a) Make a list of all weekly commitments with estimated time needed.
   b) Be sure to schedule study time. Don't forget eating, sleeping, and relaxing!

Teaching Notes:
- Most developmental students have a high anxiety level with mathematics.
- Many developmental students are hesitant to ask questions and seek extra help.
- Be sure to include your individual expectations. Keep your expectations clear and concise.

Mini-Lecture 1.2
Place Value, Names for Numbers, and Reading Tables

Learning Objectives:

1. Find the place value of a digit in a whole number.
2. Write a whole number in words and in standard form.
3. Write a whole number in expanded form.
4. Read tables.
5. Key Vocabulary: *whole numbers, place value, standard form, period, expanded form, tables.*

Examples:

1. Find the place value of the digit 7 in each whole number.

   a) 7,352   b) 607   c) 702,433   d) 17,009,321

2. Write each whole number in words.

   a) 62   b) 698   c) 17,403   d) 1,067,599

   Write each number in standard form.

   e) nine hundred fifty-two
   f) three hundred sixty-two thousand, five hundred eighty-six
   g) three million, four hundred thousand, one hundred two

3. Write each number in expanded form.

   a) 398   b) 2,907   c) 4,089,347

4. Use the following table of Number of Students Enrolled to answer the questions.

Subject	Section 1	Section 2	Section 3	Total
Basic Mathematics	23	27	19	69
Statistics	20	25	22	67

   a) How many total students are enrolled in Basic Mathematics?
   b) How many students are enrolled in Section 3 of Statistics?

Teaching Notes:

- Students who do not have English as their first language will need additional assistance learning place value vocabulary.
- Students who do not have English as their first language may use periods instead of commas in writing numbers.

Answers: 1a) thousands, b) ones, c) hundred thousands, d) million; 2a) sixty-two, b) six hundred ninety-eight, c) seventeen thousand, four hundred three, d) one million, sixty-seven thousand, five hundred ninety-nine, e) 952, f) 362,586, g) 3,400,102; 3a)300+90+8, b) 2000+900+7, c)4,000,000+80,000+9,000+300+40+7; 4a) 69, b) 22.

Mini-Lecture 1.3
Adding and Subtracting Whole Numbers, and Perimeter

Learning Objectives:

1. Add whole numbers.
2. Subtract whole numbers.
3. Find the perimeter of a polygon.
4. Solve problems by adding or subtracting whole numbers.
5. Key Vocabulary: *sum, addend, perimeter, minuend, subtrahend and difference.*

Examples:

1. Add.

   a) $3 + 9$

   b) $40 + 70$

   c) $1900 + 17$

   d) $5703 + 0$

   e) $\begin{array}{r} 51 \\ +27 \\ \hline \end{array}$

   f) $\begin{array}{r} 7329 \\ +\ 683 \\ \hline \end{array}$

   g) $93 + 145 + 69$

   h) $\begin{array}{r} 6{,}403 \\ +\ \ 793 \\ \hline 17{,}187 \end{array}$

2. Subtract. Check by adding.

   a) $11 - 7$

   b) $15 - 8$

   c) $22 - 22$

   d) $31 - 0$

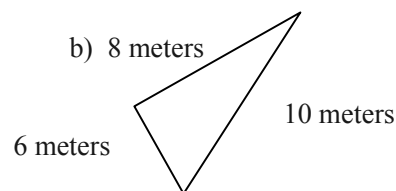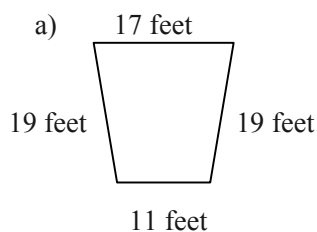   e) $\begin{array}{r} 27 \\ -\ 13 \\ \hline \end{array}$

   f) $\begin{array}{r} 198 \\ -\ \ 94 \\ \hline \end{array}$

   g) $\begin{array}{r} 3004 \\ -2965 \\ \hline \end{array}$

   h) $\begin{array}{r} 20{,}003 \\ -16{,}867 \\ \hline \end{array}$

3. Find the perimeter of each figure

   a) 17 feet

   19 feet   19 feet

   11 feet

   b) 8 meters

   6 meters   10 meters

4. Solve the following word problems.
   a) What is the sum of 8,932 and 14,799?

   b) Subtract 376 from 803.

   c) The Library Renovation Project has set a goal of $75,000 to fundraise. To date, $47,908 has been fundraised. How much more money does the Library Renovation Project need to fundraise?

   d) On Monday, Karen drove 57 miles; on Tuesday, she drove 39 miles; and on Wednesday, Karen drove 92 miles. How many total miles did Karen drive?

Teaching Notes:
- Some students need additional practice with basic addition and subtraction facts.
- Remind students that it is acceptable to write the carry digit in order to obtain the correct answer.
- Most students will find this section easy but may need assistance with word problems.
- Many students need to write the borrowing/regrouping step to maintain accuracy.
- Many students are challenged when borrowing with zeros.

Answers: 1a) 12, b) 110, c) 1917, d) 5703, e) 78, f) 8012, g) 307, h) 24,383; 2a)4, b) 7, c) 0, d) 31, e) 14, f) 104, g) 39, h) 3,136; 3a) 66 ft., b) 24 m.; 4a) 23,731, b) 427, c) $27,092, d) 188 miles.

Mini-Lecture 1.4
Rounding and Estimating

Learning Objectives:

1. Round whole numbers.
2. Use rounding to estimate sums and differences.
3. Solve problems by estimating.
4. Key Vocabulary: *rounding, graph, estimating, and exact.*

Examples:

1. Round to the nearest ten.

   a) 31   b) 57   c) 346   d) 2,795

   Round to the nearest hundred.

   e) 312   f) 6,658   g) 8,672   h) 1,899

2. Round to the nearest thousand to find the estimated sum or difference.

a)	b)	c)	d)
4892	2731	17,032	24,803
-2305	+ 3020	- 12,513	+ 14,587

3. Solve.
   a) At the last 3 dances, attendance was 657 students, 403 students, and 559 students. Estimate the total attendance by rounding each to the nearest hundred.

   b) Enrollment figures at the Town of Johnson's School Department increased from 6,721 students to 7,653 students. Round each number to the nearest hundred to estimate the increase.

   c) The Carlisle family needs to buy a refrigerator for $999, a stove for $459, and a dishwasher for $449. Round each cost to the nearest hundred to estimate the total cost.

Teaching Notes:

- Some students need to be repeatedly reminded to look at the digit to the right of the rounding position. Have students draw a line after the digit in the rounding position.
- A common error students make is to leave the digits to the right of the rounding position the same instead of changing them to zeros after rounding.
- Stress the importance of rounding and estimating with applications.

Answers: 1a) 30, b) 60, c) 350, d) 2,800, e) 300, f) 6,700, g) 8,700, h) 1,900; 2a) 3000, b) 6000, c) 4000, d) 40,000; 3a) 1700, b) 1,000, c) $1,900

Mini-Lecture 1.5
Multiplying Whole Numbers and Area

Learning Objectives:

1. Use the properties of multiplication.
2. Multiply whole numbers.
3. Find the area of a rectangle.
4. Solve problems by multiplying whole numbers.
5. Key Vocabulary: *factor, product, distribute, and area.*

Examples:

1. Multiply.

   a) $37 \cdot 1$ b) $1 \cdot 22$ c) $0 \cdot 183$ d) $9 \cdot 5 \cdot 0$

   Use the distributive property to rewrite each expression.

   e) $2(5+4)$ f) $5(1+9)$ g) $10(9+6)$ h) $15(0+14)$

2. Multiply.

   a)   37   b)   412 x 4   c)   1708   d)   337
      x 6   x   9   x 25

   e) 643   f)   309   g)   825 x 1,000   h)   2,477
      x 27   x 800   x 963

3. Find the area of a rectangle with length 14 feet and width 8 feet.

4. At a recent football game, 413 adult tickets were sold at a price of $5 each. There were 127 child tickets sold at a price of $3 each. How much total amount of money in ticket sales for the game?

Teaching Notes:

- Some students need additional practice with basic multiplication facts.
- Some students do not know the different types of symbols used for multiplication.
- When using distributive property, many students forget to distribute over both terms.
- When multiplying, remind students to carefully line up the ones, tens, hundreds, etc.

Answers: 1a) 37, b) 22, c) 0, d) 0, e) 18, f) 50, g) 150, h) 210; 2a) 222, b) 1648, c) 15,372, d) 8,425, e) 17,361, f) 247,200, g) 825,000, h) 2,385,351; 3) 112 sq .ft.; 4) $2,446

Mini-Lecture 1.6
Dividing Whole Numbers

Learning Objectives:

1. Divide whole numbers
2. Perform long division.
3. Solve problems that require dividing by whole numbers.
4. Find the average of a list of numbers
5. Key Vocabulary: *dividend, divisor, quotient, and average.*

Examples:

1. Find each quotient. Check by multiplying.

    a) $3\overline{)12}$
    b) $13 \div 1$
    c) $\dfrac{5}{5}$
    d) $15 \div 15$
    e) $0\overline{)5}$

2. Divide. Check by multiplying.

    a) $228 \div 4$
    b) $\dfrac{572}{7}$
    c) $1570 \div 3$

    d) $14\overline{)7070}$
    e) $97\overline{)41,270}$
    f) $603\overline{)604,911}$

3. a) Find the quotient of 94 and 5.

    b) Recently, Amy earned $1,722 selling calendars. If each calendar cost $14, how many calendars did Amy sell?

4. a) During the semester, Kyle's test scores were: 87, 93, 62, 83 and 100. What was Kyle's average for the semester?

Teaching Notes:

- Some students need additional practice with basic division facts.
- Many students confuse division by zero (undefined) and zero divided by any non-zero number (= 0).
- Many students need to be cautious with placement of digits in quotient and dividend. Be sure appropriate place values are lined up. Stress organization!

Answers: 1a) 4, b) 13, c) 1, d) 1, e) undefined; 2a) 57, b) 81r5, c) 523 r1; d) 505, e) 425 r45, f) 1003 r102; 3a) 18 r4, b) 123; 4a) 85

Mini-Lecture 1.7
Exponents and Order of Operations

Learning Objectives:

1. Write repeated factors using exponential notation.
2. Evaluate expressions containing exponents.
3. Use the order of operations.
4. Find the area of a square.
5. Key Vocabulary: *exponential notation, exponent, base, order of operations, and area of a square.*

Examples:

1. Write using exponential notation.

   a) $2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2$ b) $(7)(7)(7)$ c) $4 \cdot 4 \cdot 3 \cdot 3 \cdot 3$ d) $5 \cdot 5 \cdot 8 \cdot 8 \cdot 5 \cdot 5$

2. Evaluate.

   a) 5^2 b) 7^3 c) 3^6 d) 10^4

3. Using order of operations, simplify.

   a) $3 \cdot 4 - 10 \div 2$ b) $6^2 \div 3 \cdot 2$ c) $8 \cdot 4 + \{27 \div [8 - (3 + 2)]\}$

4. a) Find the area of a square whose side measures 6 feet.

   b) Find the area of a square whose side measures 23 miles.

Teaching Notes:

- Students may confuse exponent and base.
- Many students have trouble with order of operations.
- Avoid "PEMDAS" as many students will multiply before dividing and add before subtracting.
- Stress to students that all multiplication/division must be preformed in order from left to right.
- Stress to students that addition/subtraction is preformed in order from left to right.
- Stress to students you can only add/subtract after all multiplication/division is complete.

Answers: 1a)2^6, b) 7^3, c) $3^3 \cdot 4^2$, d) $5^4 \cdot 8^2$; 2a) 25, b) 343, c) 729, d) 10,000; 3a) 7, b) 24, c)41; 4a)36 sq ft, b) 529 sq ft.

Mini-Lecture 1.8
Introduction to Variables, Algebraic Expressions, and Equations

Learning Objectives:

1. Evaluate algebraic expressions given replacement values.
2. Identify solutions of equations.
3. Translate phrases into variable expressions.

Addition (+)	Subtraction (-)	Multiplication (•)	Division (÷)
Sum, plus, added to, more than, increased by, total	Difference, minus, subtract, less than, decreased by, less	Product, times, multiply, multiply by, of, double, triple	Quotient, divide, shared equally among, divided by, divided into

4. Key Vocabulary: *variable, algebraic expression, equation, and solution.*

Examples:

1. Evaluate each expression for $x = 12$, $y = 4$, and $z = 3$

   a) $x - y + z$ b) $x - (y + z)$ c) $5(3x + 7)$ d) $2xy - 3z$

   e) $x^2 - 4y$ f) $y^3 - 2x$ g) $\dfrac{3x}{4} - \dfrac{yz}{3}$ h) $\left(\dfrac{2yz - x}{2}\right)^3$

2. Determine whether the given number is a solution of the given equation.
   - a) Is 10 a solution of n – 3 = 7?
   - b) Is 3 a solution of 2n = 12?

   Determine which numbers in each set are solutions to the corresponding equations.
   - c) n – 3 = 12; {11, 12, 15}
   - d) 4n = 24; {3, 6, 20}

3. Write each phrase as a variable expression. Use x to represent "a number."

   a) The sum of a number and eleven
   b) Fifteen added to a number
   c) The difference between a number and three hundred
   d) A number subtracted from forty-two
   e) The product of sixteen and a number
   f) A number times thirteen
   g) The quotient of thirty and a number
   h) Seven divided by a number
   i) The quotient of eighteen and a number, decreased by two

Teaching Notes:

- Remind students that order of operations apply with variables.
- Stress to students that an equation has an equal sign and an expression does not.
- Many students will have difficulty translating a phrase into an algebraic expression.
- Refer students to textbook for ***Translating Phrases into Variable Expressions Chart.***

Answers: 1a) 11, b) 5, c) 215, d) 87, e) 128, f) 40, g) 5, h)216; 2a)yes, b) no, c) 15, d) 6; 3a) x+11, b) 15+x, c)x – 300, d) 42-x, e) 16x, f) 13x, g) 30/x, h) 7/x, i) $\dfrac{18}{x} - 2$

Mini-Lecture 2.1
Introduction to Integers

Learning Objectives:

1. Represent real-life situations with integers.
2. Graph integers on a number line.
3. Compare integers.
4. Find the absolute value of a number.
5. Find the opposite of a number.
6. Read bar graphs containing integers.
7. Key Vocabulary: *positive numbers, negative numbers, signed numbers, integers, is less than, is greater than, opposite, absolute value.*

Examples:

1. Represent each quantity by an integer.

    a) A scuba diver is swimming 25 feet below sea level.
    b) The record high temperature for the town is 113°F.
    c) The number of televisions sold reflected a 35 percent loss from the previous year.

2. Graph each integer in the list on the same number line.

    a) 1, 3, 5, 6   b) 2, -2, 3, -3   c) 4, 0, -2, -5   d) 0, -1, -2, -5

3. Insert < or > between each pair of integers to make a true statement.

    a) 5 ____ 10   b) 0 ____ −3   c) −42 ____ −38   d) −22 ____ 22

4. Simplify.

    a) $|2|$ b) $|-12|$ c) $|-3|$ d) $-|14|$

    e) $-|45|$ f) $-|-103|$ g) $|-x|$ if $x = -25$ h) $|x|$ if $x = -8$

5. Find the opposite of each integer.

    a) 9   b) −15   c) 0   d) −16

6. The bar graph shows the January temperatures for four days in Boston..
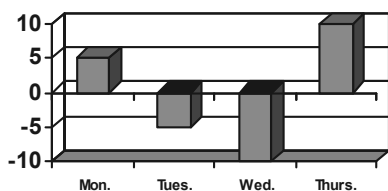
    a) Which day was the coldest?

    b) Which day was the warmest?

Teaching Notes:

- Many students will confuse absolute value and opposite.
- Encourage students to list everyday situations where negative numbers are used.

Answers: 1a) -25; b) +113; c) −35; 2a) *b)* *c)*

d) *; 3a) <; b) >; c) <; d) <; 4a) 2, b) 12; c) 3; d) -14; e) -45; f) -103;*

g) 25; h)8; 5a) -9; b) 15; c) 0; d) 16; 6a) Wed.; b) Thurs.

Mini-Lecture 2.2
Adding Integers

Learning Objectives:

1. Add integers.
2. Evaluate an algebraic expression by adding.
3. Solve problems by adding integers.

Examples:

1. Add.

   a) $23 + 12$

   b) $-23 + (-17)$

   c) $-11 + (-2)$

   d) $-21 + (-13)$

   e) $6 + (-8)$

   f) $-3 + 5$

   g) $-74 + 27$

   h) $-51 + (24)$

   i) $-8 + (-13)$

   j) $-79 + 97$

   k) $46 + (-54)$

   l) $-4 + (-24)$

   m) $23 + (-19) + (-8)$

   n) $14 + 25 + (-16)$

   o) $-25 + (-4) + (-2) + (-6)$

2. Evaluate $x + y$ for the given replacement values.

   a) $x = -5$ and $y = 14$

   b) $x = -33$ and $y = -27$

   c) $x = -43$ and $y = 38$

3. Solve.

   a) Find the sum of -7 and 25.

   b) Find the sum of $-52, \ 13, \ and -82$

   c) During a storm in Anchorage Alaska, the temperature was 10°F at Noon. At 1 p.m., the temperature had dropped 7°. At 2 p.m., the temperature dropped another 5°; and finally, at 3 p.m., the temperature had dropped an additional 9°. Use positive and negative numbers to represent his situation. Then find the present temperature.

Teaching Notes:

- Some students need to see adding integers done on a number line first.
- Many students have a better understanding if they think of depositing and withdrawing money from a bank account.
- Refer students to the rules for adding signed numbers in the textbook.

Answers: 1a) 35; b) –40; c) –13; d) –34; e) –2; f) 2; g) –47; h) –27; i) –21; j)18; k) –8; l) –28; m) –4; n) 23; o) –37; 2a) 9; b) –60; c) –5; 3a) 18; b) –121; c) -11°.

Mini-Lecture 2.3
Subtracting Integers

Learning Objectives:

1. Subtract integers.
2. Add and subtract integers.
3. Evaluate an algebraic expression by subtracting.
4. Solve problems by subtracting integers.
5. Key Vocabulary: *additive inverse.*

Examples:

1. Subtract.

   a) $-9-(-2)$ b) $-14-(-2)$ c) $4-(-3)$ d) $20-20$

   e) $2-5$ f) $-2-12$ g) $-150-410$ h) $-147-(-85)$

2. Simplify.

   a) $6+20-15$ b) $-1-11-12$ c) $-1-20+10$ d) $-16+11-18+(-4)$

3. Evaluate $x-y$ for the given replacement values.

   a) $x=-2$ and $y=-8$ b) $x=8$ and $y=-32$

   c) $x=-9$ and $y=-9$ d) $x=3$ and $y=-15$

4. Solve.

   a) Amy has $545 in her checking account. She writes a check for $257, makes a deposit of $75, and then writes another check for $409. Find the balance in her account. (Write the amount as an integer.)

   b) The city of Manchester has an elevation of 13,005 feet above sea level while the city of Catherine has an elevation of 17,532 feet below sea level. Find the difference in elevation between those two cities.

   c) The temperature on a January morning in Worcester is $-5°F$ at 2 a.m. If the temperature drops 4° by 3 a.m., rises 6° by 4 a.m., and then drops 8° by 5 a.m., find the temperature by 8 a.m.

Teaching Notes:

- Many students find subtracting signed numbers difficult at first.
- Some students like to see subtracting signed numbers on a number line.
- Many students make errors when evaluating x – y when y is a negative number. Encourage students to make a direct substitution first so they do not forget to write the subtraction symbol.

Answers: 1a) –7; b) –12; c) 7; d) 0; e) –3; f) –14; g) –560; h) –62; 2a) 11; b) –24; c) –11; d) –27; 3a) 6; b) 40; c) 0; d) 18; 4a) -$46; b) 4527 ft.; c) -11°F.

Mini-Lecture 2.4
Multiplying and Dividing Integers

Learning Objectives:

1. Multiply integers.
2. Divide integers.
3. Evaluate an algebraic expression by multiplying or dividing.
4. Solve problems by multiplying or dividing integers.

Examples:

1. Multiply.

   a) $7(-6)$

   b) $-4(10)$

   c) $-20(13)$

   d) $-10(-19)$

   e) $(-4)(-3)(6)$

   f) $(-50)(0)(-5)(8)$

   g) $(-4)(-5)(-4)(-3)$

   h) $(-2)(3)(-1)(-4)(2)$

   i) -4^2

   j) $(-3)^3$

   k) -3^3

   l) $(-8)^2$

2. Find each quotient.

   a) $21 \div 7$

   b) $36 \div (-6)$

   c) $\dfrac{-48}{6}$

   d) $\dfrac{-17}{0}$

3. Evaluate xy and also $\dfrac{x}{y}$ for the given replacement values.

   a) $x = 8$ *and* $y = -4$

   b) $x = -30$ *and* $y = -10$

   c) $x = 0$ *and* $y = -16$

4. Solve.

   a) Find the product of -13 and -5.

   b) Find the quotient of 63 and -9.

   c) Better Electric Co. marked $15 off the price of each microwave in stock. If there are 57 microwaves in stock, write the total reduction in price of all microwaves as an integer.

   d) During a cold front in Canada the temperature dropped 4°F each hour for 7 hours. Express the total drop in temperature as an integer.

Teaching Notes:

- Some students need a review of basic multiplication and division facts before they begin working with integers.
- Some students mix up the rules for addition of integers and the rules for multiplication/division of integers.
- Many students have a hard time understanding the difference between -3^2 *and* $(-3)^2$

Answers: 1a) -42; b) -40; c) -260; d)190; e) 72; f) 0; g) 240; h) -48; i) -16; j) -27; k) -27; l) 64; 2a) 3; b) -6; c) -8; d) undefined; 3a) -2; b) 3; c) 0; 4a) 65; b) -7; c) -$855; d) -28°F.

Mini-Lecture 2.5
Order of Operations

Learning Objectives:

1. Simplify expressions by using the order of operations.
2. Evaluate an algebraic expression.
3. Find the average of a list of numbers.

Examples:

1. Simplify.

   a) $-2 + 5 \cdot 6$

   b) $-2 - 5(5 - 8)$

   c) $2(-5)(7 - 3) - 7$

   d) $80 \div (-8) - 15$

   e) $3^3 - 8(2)$

   f) $8 - 2(7 - 2^2) + 3$

   g) $8^2 - 2(6) + 45 \div 5$

   h) $3(-2) + (8 - 10)^2$

   i) $21 \div \left[7 \cdot (-15 \div (-5)) \right]$

   j) $\dfrac{8(-2) - 4 + 3}{-85 \div 5}$

   k) $\dfrac{[-36 \div (-4) - 1]}{[2 - (-2)]}$

   l) $\dfrac{20(-1) - (-5)(-2)}{3[-12 \div (-3 - 3)]}$

2. Evaluate each expression for $x = -3,\ y = 6,\ and\ z = -1$.

   a) $x + y + z$

   b) $2y - 3z + x$

   c) $x^2 - y + z$

   d) $\dfrac{8x}{2y}$

   e) $5y - x^2$

   f) $x^3 + yz$

3. Find the average of each list of numbers.

   a) -20, -9, -1, 0, 4, 6 , 6

   b) -50, -30, -15, -5

Teaching Notes:

- Many students confuse the addition/subtraction rules with the multiplication/division rules when working with many operations in one expression.
- Encourage students to perform one operation at a time.
- Refer students to ***Order of Operations*** in the textbook.

Answers: 1a)2, b)13, c) -47, d) -25, e)11, f) 5, g) 61, h) -2, i) 1, j) 1, k) 2, l) –5 ; 2a) 2, b) 12, c) 2, d) -2, e) 21, f)-31; 3a) -2, b) -25

Mini-Lecture 2.6
Solving Equations: The Addition and Multiplication Properties

Learning Objectives:

1. Identify solutions of equations.
2. Use the addition property of equality to solve equations.
3. Use the multiplication property of equality to solve equations.
4. Key Vocabulary: *equation, expression, multiplication, solution, addition and equivalent.*

Examples:

1. Decide whether the given number is a solution of the given equation.

    a) Is 12 a solution of $x + 3 = 15$

    b) Is 8 a solution of $z - 15 = 23$

    c) Is -2 a solution of $4k = k - 6$

    d) Is 5 a solution of $6(x - 2) = 3x + 1$

    e) Is $\frac{1}{2}$ a solution of $-3x = 5x + 1$

    f) Is -2 a solution of $-2x + 5 = 6x - 5x + 7$

2. Solve. Check each solution.

    a) $a + 7 = 25$

    b) $d - 4 = -19$

    c) $10z = 9z - 13$

    d) $-14 = 15 + x$

3. Solve. Check each solution.

    a) $3x = 18$

    b) $\dfrac{x}{-5} = 5$

    c) $-5y = 0$

    d) $-20x = -20$

Teaching Notes:

*   Encourage students to write down all steps in a neat, organized manner. This habit will help students as equations increase in difficulty.
*   Encourage students to use the addition property in such a way that the variable ends up with a positive coefficient.
*   Mention to students that it does not matter on which side of the equation you isolate the variable.
*   Remind students to always check their final answer by substituting it back into the original equation.

Answers: *1a) yes, b) no, c) yes, d) no, e) no, f) no; 2a) 18, b)-15, c) -13, d)-29; 3a) 6, b)-25, c) 0, d) 1*

Mini-Lecture 3.1
Simplifying Algebraic Expressions

Learning Objectives:

1. Use properties of numbers to combine like terms.
2. Use properties of numbers to multiply expressions.
3. Simplify expressions by multiplying and then combining like terms.
4. Find the perimeter and area of figures.
5. Key Vocabulary: *algebraic expression, constant, variable, numerical coefficient, like terms, distributive property, and simplify.*

Examples:

1. Simplify each expression by combining like terms.

   a) $3x + 5x$ 　　　　 b) $11y - 9y$ 　　　　 c) $5a - 19a$

   d) $6z + 15z - 5z + 7$ 　　 e) $4.2 + 8.7x - 1.9 - 3.3x$ 　　 f) $\dfrac{4}{5}x - \dfrac{2}{3} + \dfrac{1}{3}x - \dfrac{1}{5}$

2. Multiply.

   a) $7(4x)$ 　　　 b) $-12(6a)$ 　　　 c) $\dfrac{2}{5}(-15x)$ 　　　 d) $-5(3y - 2)$

3. Simplify each expression. Use the distributive property to remove parentheses.

   a) $5(y + 3) - 6$ 　　　 b) $2(7 - 3a) + a$ 　　　 c) $-3(x + 1) + 4(8 - x) - 18$

4. Find the perimeter of the figures.

   a) 5 in 　 5 in 　 6 in 　　　　 b) 6 ft 　 12 ft
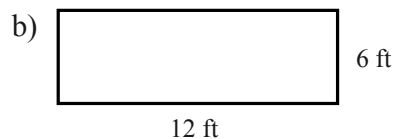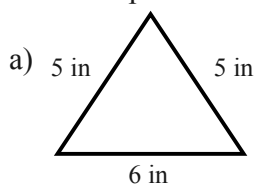
Teaching Notes:

- Some students need to practice identifying "like terms".
- Some students do not know that a variable without a numerical coefficient actually has a coefficient of 1.
- Many students tend to make careless errors associated with the distributive property.
- Remind students that perimeter is the distance around an object.

Answers: 1a) 8x, b) 2y, c) –14a, d) 16z + 7, e)5.4x + 2.3, f)17/15 x – 13/15; 2a) 28x, b) –72a, c) –6x, d) -15y +10; 3a) 5y + 9, b) 14 – 5a, c) –7x + 11; 4a) 16in, b) 36 ft

Mini-Lecture 3.2
Solving Equations: Review of the Addition and Multiplication Properties

Learning Objectives:

1. Use the addition property or the multiplication property to solve equations.
2. Use both properties to solve equations.
3. Translate word phrases to mathematical expressions.
4. Key Vocabulary: *expression, and equation*

Examples:

1. Solve. Check each solution.

   a) $-14 = 15 + x$

   b) $3y - 7y = 12$

   c) $\dfrac{x}{4} = 11 - 5$

   d) $5x + 2 - 4x = 7 - 19$

   e) $3(3x - 5) = 10x$

   f) $13x = 4(3x - 1)$

2. Solve. Both properties.

   a) $5y + 2 = 17$

   b) $3x - 5 = 10$

   c) $-4(x + 2) - 60 = -8$

   d) $9 - 14 = \dfrac{x}{-12}$

3. Write each phrase as a variable expression. Use x to represent "a number."
   a) Eight subtracted from a number
   b) The product of a number and 5
   c) The quotient of a number and negative 7
   d) The total of twice a number and 3

Teaching Notes:

- Encourage students to write down all steps in a neat, organized manner. This habit will help students as equations increase in difficulty.
- Encourage students to use the addition property in such a way that the variable ends up with a positive coefficient.
- Mention to students that it does not matter on which side of the equation you isolate the variable.
- Remind students to always check their final answer by substituting it back into the original equation.

Answers: 1a) -29, b)-3, c)24, d)-14, e)-15, f)-4; 2a)3, b)152, c) -15, d)60; 3a)x – 8, b)5x, c)x/-7, d)2x + 3

Mini-Lecture 3.3
Solving Linear Equations in One Variable

Learning Objectives:

1. Solve linear equations using the addition and multiplication properties.
2. Solve linear equations containing parentheses.
3. Write numerical sentences as equations.

Examples:

1. Solve each equation. Remember to check your answer by substitution.

   a) $2x - 20 = 0$
   b) $3p + 5 = 4p + 11$
   c) $6y + 21 = 5y + 9$

   d) $10z = 7z + 10 + 2z$
   e) $-2a + 24 = -8a - 6a$
   f) $40 - 5y + 5 = -2y - 10 - 4y$

2. Solve each equation. Remember to check your answer by substitution.

   a) $5(y + 5) = 6(y - 8)$ 　　　　　　　　b) $3(y + 5) = 4(y - 4)$

   c) $4(2x - 4) = 7(x + 5)$ 　　　　　　　　d) $6(2a - 3) = 9(a + 4)$

   e) $-7y + 6(-3y - 7) = -64 - 3y$ 　　　　f) $6b + 5(-3b - 2) = -12 - 7b$

   g) $-2(8y - 6) - 2(-7y - 3) = -8$ 　　　　h) $5(2z - 2) = 9(z + 5)$

3. Write each sentence as an equation.

   a) The sum of –57 and 49 is –8.

   b) The difference of negative 31 and 15 is negative 46.

   c) The quotient of –10 and 2 amounts to –5.

Teaching Notes:
- Encourage students to write out each step rather than doing it in their head.
- Remind students that it does not matter which side you isolate the variable.
- Caution students to take their time using the distributive property.
- Refer students to the textbook for **Steps for Solving an Equation**.
- Refer students to the textbook for **Key Words or Phrases** chart.
- Remind students to always check their final answer by substituting it back into the original equation.

Answers: 1a) 10, b) –6, c) –12, d) 10, e) –2, f) -55; 2a) 73, b) 31, c) 51, d) 18, e) 1, f) 1, g) 13, h) 55; 3a) –57 + 49 = -8, b) –31 – 15 = -46, c) –10/2 = -5.

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Mini-Lecture 3.4
Linear Equations in One Variable and Problem Solving

Learning Objectives:

1. Write sentences as equations.
2. Use problem-solving steps to solve problems.
3. Key Vocabulary: *sentence → equation.*

Examples:

1. Write each sentence as an equation. Use x to represent "a number". Do not solve.

   a) A number added to –12 equals 15.

   b) Two subtracted from a number amounts to 55.

   c) Ten subtracted from ten times a number is equal to 150.

   d) The product of a number and -4 is twice the sum of the number and 2.

   e) The quotient of 10 and a number is 130.

2. Translate each to an equation and solve the resulting equation.

   a) Six times a number yields 36. Find the number.

   b) A number subtracted from 16 amounts to the quotient of 42 and 6. Find the number.

   c) The difference of –6 times some number and 12 gives –8 times the sum of the number and –8. Find the number.

   d) A Ford Taurus is traveling three times as fast as a Honda CRV. If their combined speed is 96 miles per hour, find the speed of each car.

Teaching Notes:

- Refer students to **Key Words and Phrases** chart.
- Refer students to **Problem-Solving Steps** chart.
- Remind students that a phrase is translated into an expression; a sentence is translated into an equation.
- Many students have difficulty translating words into mathematical symbols. This section will be a challenge to most students.

Answers: 1a) x + (-12) = 15, b) x – 2 = 55, c) 10x – 10 = 150, d) -4x=2(x+2), e) 10x=130; 2a) 6x + 36, x = 6,
b) 16 – x = 42/6, x = 9, c) –6x – 12 = -8[x + (-8)], x = 38, d) x + 3x = 96; Ford = 72 mph, Honda = 24 mph.

Mini-Lecture 4.1
Introduction to Fractions and Mixed Numbers

Learning Objectives:
1. Identify the numerator and the denominator of a fraction.
2. Write a fraction to represent parts of figures or real-life data.
3. Graph fractions on a number line.
4. Review division properties of 0 and 1.
5. Write mixed numbers as improper fractions.
6. Write improper fractions as mixed numbers or whole numbers.
7. Key Vocabulary: *fractions, numerator, denominator, proper fraction, improper fraction, mixed number.*

Examples:
1. Identify the numerator and the denominator of a fraction.

   a) $\dfrac{3}{7}$ b) $\dfrac{12}{13}$ c) $\dfrac{10}{7}$ d) $\dfrac{13}{13}$

2. Represent the shaded part and unshaded part of each figure as a fraction..

   a)   b)
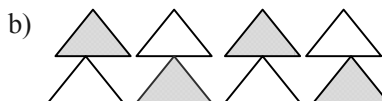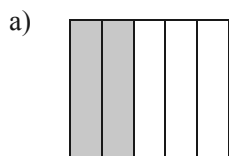
   Draw and shade a part of a diagram to represent each fraction.

   c) $\dfrac{1}{6}$ of a diagram   d) $\dfrac{5}{9}$ of a diagram.

   Write a fraction to represent the following information.

   e) Of the 207 students taking Basic Mathematics, 143 are freshman. What fraction of the class is freshman?

3. Graph each fraction on a number line.

   a) $\dfrac{1}{4}$ b) $\dfrac{8}{3}$ c) $\dfrac{9}{7}$ d) $\dfrac{3}{5}$

4. Simplify.

   a) $\dfrac{4}{4}$ b) $\dfrac{-7}{1}$ c) $\dfrac{0}{2}$ d) $\dfrac{-12}{0}$

5. Write each mixed number as an improper fraction.

   a) $3\dfrac{1}{2}$ b) $2\dfrac{8}{9}$ c) $13\dfrac{2}{9}$ d) $103\dfrac{3}{11}$

6. Write each improper fraction as a mixed number or a whole number.

   a) $\dfrac{16}{3}$ b) $\dfrac{38}{5}$ c) $\dfrac{156}{12}$ d) $\dfrac{159}{143}$

Teaching Notes:
- Students need to have a firm grasp of fraction vocabulary before continuing.
- Many students confuse $\dfrac{0}{x}$ with $\dfrac{x}{0}$. Be sure to stress the difference.
- Many students can write a fraction to represent a real-life situation, but they do not truly understand the meaning.

Answers: 1a) n=3, d=7; b) n=12, d=13; c) n=10, d=7; d) n=13, d=13; 2a) 2/5, 3/5; b) 4/8, 4/8;
2c – 2d) diagrams will vary; e) 143/207; 3a) – 3d) number lines; 4a) 1; b) –7; c) 0; d) undefined ; 5a) 7/2;
b) 26/9; c) 119/9; d) 1136/11; 6a) 5 1/3; b) 7 3/5; c) 13; d) 1 16/143

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Mini-Lecture 4.2
Factors and Simplest Form

Learning Objectives:

1. Write a number as a product of prime numbers.
2. Write a fraction in simplest form.
3. Determine whether two fractions are equivalent.
4. Solve problems by writing fractions in simplest form.
5. Key Vocabulary: *factor, prime factorization, prime numbers, composite number, simplest form, lowest terms.*

Examples:

1. Write the prime factorization of each number.

   a) 30   b) 75   c) 170   d) 360

2. Write each fraction in simplest form.

   a) $\dfrac{10}{16}$ b) $\dfrac{36}{63}$ c) $\dfrac{77}{88}$ d) $-\dfrac{12}{42}$

   e) $\dfrac{11}{34}$ f) $\dfrac{-27}{36}$ g) $\dfrac{30}{80}$ h) $\dfrac{6}{-105}$

3. Determine whether each pair of fractions is equivalent.

   a) $\dfrac{5}{10}$ *and* $\dfrac{11}{22}$ b) $\dfrac{7}{21}$ *and* $\dfrac{8}{24}$ c) $\dfrac{2}{7}$ *and* $\dfrac{8}{15}$ d) $\dfrac{6}{0}$ *and* $\dfrac{0}{6}$

4. Solve. Write each fraction in simplest form.

   a) Alicia was scheduled to work 6 hours at the tanning salon. What fraction of Alicia's shift is represented by 4 hours?

   b) There are 36 inches in a yard. What fraction of a yard is represent by 9 inches?

   c) There are 140 students in a freshman lecture class. If 16 students are absent, what fraction of the students are absent?

Teaching Notes:

- Many students will understand equivalent fractions if they are shown drawings.
- Some students will confuse cross products and simplifying. Stress that cross products is only a check to determine equality of fractions.
- Some students prefer to reduce fractions by factoring the numerator and denominator as products of prime numbers, then canceling all common factors. Others prefer to repeatedly divide the numerator and the denominator by a common factor.

Answers: 1a) $2 \cdot 3 \cdot 5$, b) $3 \cdot 5^2$, c) $2 \cdot 5 \cdot 17$, d) $2^3 \cdot 3^2 \cdot 5$; 2a) 5/8, b)4/7, c) 7/8, d) -2/7, e) cannot be simplified, f) -3/4, g) 3/8, d) -2/35; 3a) yes, b) yes, c) no, d) no; 4a) 2/3; b) ¼; c) 4/35

Mini-Lecture 4.3
Multiplying and Dividing Fractions

Learning Objectives:

1. Multiply fractions.
2. Evaluate exponential expressions with fractional bases.
3. Divide fractions.
4. Multiply and divide given fractional replacement values.
5. Solve applications that require multiplication of fractions.
6. Key Vocabulary: *reciprocal, "of"*.

Examples:

1. Multiply. Write the product in simplest form.

   a) $\dfrac{1}{9} \cdot \dfrac{1}{7}$
   b) $\dfrac{2}{3} \cdot \dfrac{1}{4}$
   c) $-\dfrac{5}{6} \cdot -\dfrac{2}{3}$
   d) $-\dfrac{7}{2} \cdot \dfrac{6}{3}$

   e) $\dfrac{5}{2} \cdot \dfrac{18}{15}$
   f) $\dfrac{7}{8} \cdot 0$
   g) $\dfrac{1}{2} \cdot -\dfrac{3}{5} \cdot \dfrac{1}{5}$
   h) $-\dfrac{12}{14} \cdot -\dfrac{3}{9} \cdot -\dfrac{2}{10}$

2. Evaluate.

   a) $\left(\dfrac{1}{2}\right)^2$
   b) $\left(-\dfrac{1}{3}\right)^4$
   c) $\left(-\dfrac{2}{5}\right)^3$
   d) $\left(\dfrac{2}{7}\right)^2 \cdot \dfrac{1}{4}$

3. Divide. Write all quotients in simplest form.

   a) $\dfrac{3}{5} \div \dfrac{4}{7}$
   b) $\dfrac{1}{4} \div \dfrac{1}{4}$
   c) $-\dfrac{1}{5} \div \dfrac{9}{19}$
   d) $\dfrac{8}{17} \div \dfrac{12}{15}$

   e) $-\dfrac{2}{17} \div -\dfrac{3}{17}$
   f) $\dfrac{1}{14} \div 0$
   g) $\dfrac{27}{-7} \div \dfrac{4}{7}$
   h) $0 \div \dfrac{-3}{11}$

4. Given the following replacement values, evaluate (a) xy and (b) $x \div y$.

   a) $x = -\dfrac{1}{3}$ and $y = \dfrac{4}{9}$
   b) $x = \dfrac{5}{7}$ and $y = -\dfrac{5}{9}$

5. Solve. Write each answer in simplest form.

   a) Find $\dfrac{1}{3}$ of 48.
   b) Find $\dfrac{3}{7}$ of -63

   c) A bike trail is 27 miles long. Michelle bikes $\dfrac{2}{3}$ of the trail. How many miles did Michelle bike?

Teaching Notes:

- Encourage students to divide out common factors in the numerator and denominator before multiplying.
- When dividing, encourage students take the time and rewrite the problem by changing the division symbol to multiplication and multiply by the reciprocal. Many students begin "simplifying" and forget to multiply by the reciprocal.

Answers: 1a) 1/63, b) 1/6, c) 5/9, d) -7, e) 3, f) 0, g) -3/50, h) –2/35; 2a) ¼, b) 1/81, c) -8/125, d) 1/49, 3a) 21/20, b) 1, c) –19/45, d) 10/17, e) 2/3, f) undefined, g) -27/4, h) 0; 4a) -4/27, -3/4; b) -25/63, -9/7; 5a) 16, b) -27, c) 18.

Mini-Lecture 4.4
Adding and Subtracting Like Fractions, Least Common Denominator, and Equivalent Fractions

Learning Objectives:

1. Add or subtract like fractions.
2. Add or subtract given fractional replacement values.
3. Solve problems by adding or subtracting like fractions.
4. Find the least common denominator of a list of fractions.
5. Write equivalent fractions.
6. Key Vocabulary: *like fractions, unlike fractions, (LCD) least common denominator, (LCM) least common multiple, and equivalent fractions.*

Examples:

1. Add and simplify.

   a) $\dfrac{1}{9} + \dfrac{4}{9}$

   b) $\dfrac{1}{8} + \dfrac{5}{8}$

   c) $-\dfrac{1}{10} + \dfrac{9}{10}$

   d) $-\dfrac{3}{14} + \left(-\dfrac{5}{14}\right)$

   Subtract and simplify.

   e) $\dfrac{6}{8} - \dfrac{3}{8}$

   f) $-\dfrac{6}{21} - \dfrac{5}{21}$

   g) $\dfrac{25}{42} - \left(-\dfrac{7}{42}\right)$

   h) $-\dfrac{28}{13} - \left(-\dfrac{5}{13}\right)$

2. Evaluate each expression if $x = \dfrac{3}{5}$ and $y = -\dfrac{1}{5}$.

   a) $x + y$

   b) $x \cdot y$

   c) $x - y$

   d) $x \div y$

3. Solve.

   a) Find the perimeter of a triangle with sides: $\dfrac{6}{25}$ inch, $\dfrac{9}{25}$ inch, and $\dfrac{5}{25}$ inch.

   b) Cori read $\dfrac{2}{11}$ of her book on Friday, $\dfrac{3}{11}$ of her book on Saturday, and $\dfrac{4}{11}$ of her book on Sunday. What part of her book has she read?

4. Find the LCM of each list of numbers.

   a) 10, 12

   b) 12, 14

   c) 30, 35

   d) 30, 20, 50

5. Write each fraction as an equivalent fraction with the given denominator.

   a) $\dfrac{4}{9} = \dfrac{}{18}$

   b) $\dfrac{7}{11} = \dfrac{}{55}$

   c) $\dfrac{5}{2} = \dfrac{}{6}$

   d) $\dfrac{2}{3} = \dfrac{}{24}$

Teaching Notes:

- Many students add or subtract both numerator and denominator.
- When subtracting, some students may need to take the intermediate step of writing out the operations performed on the numerators. For example: $-\dfrac{1}{7} - \left(-\dfrac{3}{7}\right) = -\dfrac{1}{7} + \left(+\dfrac{3}{7}\right) = \dfrac{-1+3}{7} = \dfrac{2}{7}$.
- Some students forget to multiply the numerator when building equivalent fractions.

Answers: 1a) 5/9, b) ¾, c) 4/5, d) -4/7, e) 3/8, f) -11/21, g) 16/21, h) -23/13; 2a) 2/5, b) -3/25, c) 4/5, d) -3; 3a) 4/5 inch, b) 9/11, 4a) 60, b) 84, c) 210, d) 300; 5a) 8, b) 35, c) 15, d) 16.

Mini-Lecture 4.5
Adding and Subtracting Unlike Fractions

Learning Objectives:

1. Add or subtract unlike fractions.
2. Write fractions in order.
3. Evaluate expressions given fractional replacement values.
4. Solve problems by adding or subtracting unlike fractions.
5. Key Vocabulary: *least common denominator (LCD)*.

Examples:

1. Add or subtract as indicated.

   a) $\dfrac{1}{10} + \dfrac{2}{5}$

   b) $-\dfrac{1}{5} + \left(-\dfrac{2}{25}\right)$

   c) $\dfrac{1}{7} + \left(-\dfrac{9}{10}\right)$

   d) $\dfrac{3}{5} + \dfrac{1}{20}$

   e) $\dfrac{4}{5} - \dfrac{3}{20}$

   f) $\dfrac{7}{9} - \left(-\dfrac{1}{12}\right)$

   g) $-\dfrac{5}{7} - \left(-\dfrac{1}{2}\right)$

   h) $\dfrac{1}{20} - \dfrac{8}{15}$

   i) $-\dfrac{7}{5} + \dfrac{8}{16} + \dfrac{4}{20}$

2. Insert < or > to form a true sentence.

   a) $\dfrac{2}{3}$ ___ $\dfrac{1}{9}$

   b) $\dfrac{5}{12}$ ___ $\dfrac{1}{2}$

   c) $-\dfrac{5}{6}$ ___ $-\dfrac{4}{5}$

3. Evaluate each expression if $x = \dfrac{1}{4}$ and $y = -\dfrac{3}{5}$.

   a) $x + y$ b) $x \cdot y$ c) $x - y$ d) $x \div y$

4. Solve.

   a) Find the perimeter of a rectangle with width $\dfrac{3}{4}$ feet and length $\dfrac{3}{14}$ feet.

   b) Sharon is making matching holiday outfits for her three children. Each outfit required $\dfrac{7}{8}$ yards.

   How many yards of material will be needed to make the three outfits?

Teaching Notes:

- Refer students back to Section 3.4: *Method 1: Finding the LCM of a List of numbers Using Multiples of the Largest Number* and *Method 2: Finding the LCM of a List of Numbers Using Prime Factorization.*
- Some students try to cross-cancel when adding or subtracting.
- Some students add and subtract both the numerator and denominator.
- Some students forget to multiply the numerator when building equivalent fractions.

Answers: 1a) ½, b) -7/25, c) -53/70, d) 13/20, e) 13/20, f) 31/36, g) –3/14, h) -29/60, i) -7/10; 2a) >, b) <, c) <; 3a) -7/20, b) -3/20, c) 17/20, d) -5/12; 4a) 27/14 feet, b) 21/8 yd

Mini-Lecture 4.6
Complex Fractions and Review of Order of Operations

Learning Objectives:

1. Simplify complex fractions.
2. Review the order of operations.
3. Evaluate expressions given replacement values.
4. Key Vocabulary: *complex fraction.*

Examples:

1. Simplify each complex fraction.

a) $\dfrac{\frac{1}{6}}{\frac{2}{3}}$

b) $\dfrac{\frac{16}{7}}{\frac{8}{7}}$

c) $\dfrac{\frac{1}{6}+\frac{1}{2}}{\frac{1}{3}+\frac{3}{4}}$

2. Use the order of operations to simplify each expression.

a) $\dfrac{1}{4}+\dfrac{1}{4}\cdot\dfrac{1}{3}$

b) $\dfrac{3}{2}\div\left(\dfrac{7}{8}+\dfrac{7}{16}\right)$

c) $\left(\dfrac{2}{7}+\dfrac{3}{14}\right)\left(\dfrac{2}{7}-\dfrac{3}{14}\right)$

d) $\left(\dfrac{2}{5}\right)^2\cdot\dfrac{1}{2}$

e) $\dfrac{1}{2}+\left(\dfrac{2}{3}\right)^2-\dfrac{1}{3}$

f) $\dfrac{2}{3}\cdot\left(\dfrac{1}{4}+\dfrac{1}{2}\right)\cdot 6$

3. Evaluate each expression if $x=-\dfrac{1}{2}$, $y=\dfrac{3}{5}$, and $z=\dfrac{7}{10}$.

a) $3x-z$

b) $\dfrac{y}{z}$

c) x^2+2y

d) $\dfrac{x+y}{z}$

Teaching Notes:

- Many students make careless errors when using Method 2 for simplifying complex fractions. If this is the case, encourage students to use Method 1 (rewrite as a division problem).
- Remind students that when dividing fractions, you must change division to multiplication and multiply by the reciprocal.
- Some students will try to apply procedures for simplifying complex fractions to adding and subtracting fractions.

Answers: 1a) ¼, b) 2, c) 8/13; 2a) 1/3, b) 8/7, c) 1/28, d) 2/25, e) 11/18, f) 3; 3a) -11/5, b) 6/7, c) 29/20, d) 1/7

Mini-Lecture 4.7
Operations on Mixed Numbers

Learning Objectives:

1. Graph positive and negative fractions and mixed numbers.
2. Multiply or divide mixed or whole numbers.
3. Add or subtract mixed numbers.
4. Solve problems containing mixed numbers.
5. Perform operations on negative mixed numbers.

Examples:

1. Graph each list of numbers on a number line.

   a) $-3, -3\frac{1}{2}, -1, \frac{3}{4}, 2$

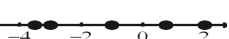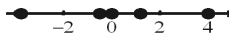   b) $4, -3\frac{3}{4}, 0, 1\frac{1}{5}, -\frac{1}{2}$

2. Multiply or divide.

   a) $2\frac{2}{3} \cdot \frac{1}{2}$

   b) $3\frac{3}{4} \cdot 1\frac{3}{5}$

   c) $4\frac{1}{5} \div \frac{1}{5}$

   d) $3\frac{1}{3} \div 2\frac{3}{5}$

3. Add or subtract.

   a) $10\frac{1}{2} + 7\frac{1}{9}$

   b) $6\frac{1}{3} + 13\frac{4}{9}$

   c) $4\frac{2}{3} + 9\frac{7}{9}$

   d) $8\frac{1}{3} + 2\frac{2}{3} + 3\frac{2}{9}$

   e) $15\frac{8}{9} - 6\frac{2}{9}$

   f) $19\frac{1}{25} - 7\frac{1}{5}$

   g) $17\frac{1}{6} - 5\frac{13}{24}$

   h) $13 - 6\frac{5}{9}$

4. Solve. Write answer in simplest form.

   a) Amy rode her bicycle $9\frac{4}{15}$ miles on each of 9 days. What is the total distance Amy rode?

   b) John cuts a board $13\frac{3}{7}$ feet long from one 20 feet long. How long is the remaining piece?

5. Perform the indicated operation.

   a) $-5\frac{7}{8} \div 5\frac{1}{4}$

   b) $-7\frac{7}{9} + \left(-4\frac{2}{9}\right)$

   c) $-8\frac{1}{3} \cdot \left(-1\frac{1}{5}\right)$

   d) $5\frac{5}{8} \div (-9)$

   e) $19\frac{3}{5} - \left(-5\frac{18}{20}\right)$

   f) $\left(-15\frac{3}{7}\right) + 14\frac{1}{5}$

   g) $10\frac{1}{9} + \left(-5\frac{5}{9}\right)$

   h) $-9 \cdot \left(5\frac{7}{12}\right)$

Teaching Notes:

- Most students forget that mixed numbers must be changed to improper fractions before multiplying. Some try to multiply the whole number parts together, and then multiply the fractional parts together.
- Many students confuse the rules for multiplication with adding/subtracting rules.
- Many students are challenged by word problems. Students have trouble deciding which operation to use for the word problems.

Answers: 1a) *1b)* *2a) 4/3, b) 6, c) 21, d) 1 11/3;*

3a) 17 11/18; b) 19 7/9; c) 14 4/9; d) 14 2/9; e) 9 2/3; f) 11 21/25; g) 11 5/8; h) 6 4/9; 4a) 83 2/5; b) 6 4/7; 5a) -47/42; b) -12; c) 10; d) -5/8; e) 25 ½; f) -1 8/35; g) 4 5/9; h) -50 ¼

Mini-Lecture 4.8
Solving Equations Containing Fractions

<u>Learning Objectives:</u>

1. Solve equations containing fractions.
2. Solve equations by multiplying by the LCD.
3. Review adding and subtracting fractions.

Examples:

1. Solve each equation. Check your proposed solution.

   a. $x + \frac{1}{2} = -\frac{1}{2} =$ b. $z - \frac{4}{15} = \frac{3}{15}$ c. $-\frac{3}{8} = x - \frac{5}{6}$

   d. $8x - \frac{3}{5} - 7x = \frac{11}{15}$

2. Solve each equation.

   a. $\frac{1}{3}x = 5$ b. $-\frac{3}{8}x = -\frac{4}{5}$ c. $-8a = \frac{16}{25}$

   d. $\frac{3}{5}y = -\frac{7}{25}$ e. $\frac{x}{7} - x = -5$ f. $\frac{b}{3} = \frac{b}{5} + \frac{7}{3}$

3. Add or subtract as indicated.

   a. $\frac{n}{2} + \frac{4}{7}$ b. $\frac{5c}{8} - \frac{c}{4}$

Teaching Notes:
- Emphasize checking proposed solutions.
- When adding or subtracting fractions, the denominators need to be the same.
- Review properties: Addition Property of Equality
                        Multiplication Property of Equality
- Make sure students understand the difference between solving an equation containing fractions (multiply both sides of the equation by the LCD of the fractions) and adding or subtracting two fractions (create equivalent fractions).

Answers: 1a) -1, b)$\frac{7}{15}$, c)$\frac{11}{24}$, d)$\frac{4}{3}$;   2a) 15, b)$\frac{32}{15}$, c) $-\frac{2}{25}$; d)$-\frac{7}{15}$, e)$\frac{35}{6}$, f) $\frac{35}{2}$;   3a) $\frac{7n+8}{14}$, b)$\frac{3c}{8}$

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Mini-Lecture 5.1
Introduction to Decimals

Learning Objectives:

1. Know the meaning of place value for a decimal number and write decimals in words.
2. Write decimals in standard form.
3. Write decimals as fractions.
4. Compare decimals.
5. Round decimals to given place values.
6. Key Vocabulary: *decimals, standard form, < (less than), > (greater than), = (equal to), round the decimal part.*

Examples:

1. Determine the place value for the digit 9 in each number.
   a) 90   b) 900   c) 0.9   d) 0.09

   Write each decimal number in words.
   e) 8.54   f) - 0.382   g) 7002.09

2. Write each decimal number in standard form.
   a) two and seven tenths   b) negative eleven and five hundredths
   c) seven hundred three and two   d) negative ninety-five ten thousandths
       hundred fifty-five thousandths

3. Write each decimal as a fraction or a mixed number. Write your answer in simplest form.
   a) 0.7   b) - 0.35   c) 0.094   d) - 2.4005

4. Insert <, >, or = to form a true statement.
   a) 0.2 ____ 0.5   b) 0.14 ___ - 0.14000
   c) 0.6401 ___ 0.6410   d) - 15.0037 ___ 15.00037

5. Round each decimal to the given place value.
   a) 0.39 to the nearest tenth   b) - 0.174 to the nearest hundredth
   c) 1.4782 to the nearest thousandth d) - 22.099 to the nearest hundredth

   Round each monetary amount to the nearest cent or dollar as indicated.
   e) $0.058 to the nearest cent   f) $17.88 to the nearest dollar

Teaching Notes:

- Most students find problems 1 and 2 easy.
- Some students have difficulty with example 3 when a whole number is involved.
- Some students become confused when rounding monetary values. When rounding to the nearest cent, it is important to remind them that this is the hundredths position (one-hundredths-of-a-dollar position).

Answers: 1a) tens, b) hundreds, c) tenths, d) hundredths, e) eight and fifty-four hundredths, f) negative three hundred eighty-two thousandths, g) seven thousand two and nine hundredths; 2a) 2.7, b) -11.05, c) 703.255, d) -0.0095;

3a) $\frac{7}{10}$, b) $-\frac{7}{20}$, c) $\frac{47}{500}$, d) $-2\frac{801}{2000}$; 4a) <, b) >, c) <, d) <; 5a) 0.4, b) -0.17, c) 1.478, d) -22.10, e) $0.06, f) $18

Mini-Lecture 5.2
Adding and Subtracting Decimals

Learning Objectives:

1. Add or subtract decimals.
2. Estimate when adding or subtracting decimals.
3. Evaluate expressions with decimal replacement values.
4. Simplify expressions containing decimals.
5. Solve problems that involve adding and subtracting decimals.

Examples:

1. Add. Be sure to estimate to see if the answer is reasonable.

   a) $0.5 + 0.1$ b) $-2.7 + -3.2$ c) $7.2 + 3.27$

   d) $-372 + 9.302$ e) $43.097 + 289.3887$ f) $5.03 + 16.988 + 0.006$

2. Subtract. Be sure to estimate to see if the answer is reasonable.

   a) $0.8 - 0.2$ b) $-7.5 - 2.3$ c) $187.5 - 8.39$

   d) $8.2 - 5.006$ e) $-632.021 - (-295.9)$ f) $1000 - 3.0947$

3. Evaluate each expression for $x = 2.4$, $y = 3$, and $z = 0.51$.

   a) $x + z$ b) $y - x$ c) $x + y - z$

4. Solve.
   a) Recently, Allison went shopping and spent $18.92 at the bookstore, $68.03 at the grocery store, and $129.76 at a department store. What is the total amount of money Allison spent?
   b) Find the perimeter of a rectangular lawn that measures 40.93 feet by 27.09 feet.

Teaching Notes:

- Remind students to work in a vertical format and line-up the decimal point and corresponding place values.
- Some students need to be shown how to add extra zeros to the ends of the decimal part of the numbers and where to place the decimal point with whole numbers.
- Some students must be reminded of how to borrow across zeros when subtracting.
- Refer students to the **Adding or Subtracting** decimals chart in the textbook.

Answers: 1a) 0.6, b) -5.9, c) 10.47, d) -362.698, e) 332.4857, f) 22.024; 2a) 0.6, b) -9.8, c) 179.11, d) 3.194, e) -336.121, f) 996.9053; 3a) 2.91, b) 0.6, c) 4.89; 4a) $216.71, b) 136.04 ft

Mini-Lecture 5.3
Multiplying Decimals and Circumference of a Circle

Learning Objectives:
1. Multiply decimals.
2. Estimate when multiplying decimals.
3. Multiply decimals by powers of 10.
4. Evaluate expressions with decimal replacement values.
5. Find the circumference of circles.
6. Solve problems by multiplying decimals.
7. Key Vocabulary: л *(pi), perimeter, circumference, diameter, radius.*

Examples:

1. Multiply.
   a) 0.7×0.2
   b) 1.33×-0.5
   c) 7.2×5.8

   d) $\begin{array}{r} 0.856 \\ \times \quad 3.1 \\ \hline \end{array}$
   e) $\begin{array}{r} -2.00033 \\ \times \quad -6.9 \\ \hline \end{array}$
   f) $\begin{array}{r} 0.0896 \\ \times \quad 0.345 \\ \hline \end{array}$

2. Multiply. Check by estimating.
   a) $(6.8)(3.2)$
   b) $(8.4)(1.8)$
   c) $(5.8)(0.7)$

3. Multiply.

   a) 4.3×10
   b) 17.693×100
   c) -0.0027×1000

   d) -0.07×-0.1
   e) 9.07×0.01
   f) 2.908×0.001

4. Evaluate each expressions for $x = 2$, $y = -0.3$, and $z = 7.3$.

   a) xy
   b) $xz - y$
   c) $-3y + z$

5. Find the circumference of a circle with the given information. Use $\pi = 3.14$.

   a) radius = 7 feet
   b) diameter = 16 inches
   c) radius = 10.3 meters

6. a) Write 57.6 million in standard form.
   b) A 1-ounce serving of hot cocoa contains 0.375 grams of fat . How many grams of fat are in an 8 oz. mug of hot cocoa?

Teaching Notes:
- Some students do not see the pattern that develops when multiplying by powers of 10, they must be shown.
- Many students prefer to multiply numbers by a power of ten the long way.
- Some students will attempt to line up the decimal point (like adding) when multiplying.
- Refer students to $C = 2\pi r$.

Answers: 1a) 0.14, b) -0.665, c) 41.76, d) 2.6536, e) 13.802277, f) 0.030912; 2a) 21.76; b) 15.12; c) 4.06; 3a) 43, b) 1769.3, c) -2.7, d) 0.007, e) 0.0907, f) 0.002908; 4a) -0.6, b) -6.7, c) 8.2; 5a) 43.96 ft. b) 50.24 in., c) 64.684 m; 6a) 57,600,000; b) 3g

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Mini-Lecture 5.4
Dividing Decimals

Learning Objectives:
1. Divide decimals.
2. Estimate when dividing decimals.
3. Divide decimals by powers of 10.
4. Evaluate expressions with decimal replacement values.
5. Solve problems by dividing decimals.

Examples:
1. Divide.

   a) $1.5 \div 5$ b) $26\overline{)7.826}$ c) $-518\overline{)0.9324}$

   d) $8.9\overline{)22.25}$ e) $-1411.51 \div -36.1$ f) $0.02\overline{)0.8}$

2. Divide. Then estimate to see if your answer is reasonable.
   a) $2.5\overline{)18.5}$ b) $2.4\overline{)35.4}$

3. Divide decimals by powers of 10.

   a) $\dfrac{7.74}{10}$ b) $1000\overline{)-887.73}$ c) $1.047 \div 100$

4. Evaluate each expression for $x = 3.02$, $y = -0.3$, and $z = 1.51$.

   a) $z \div y$ b) $z \div x$ c) $x \div y$

5. Solve.

   a) Divide 0.894 by -0.041 and round the quotient to the nearest hundredth.

   b) Preparing for a picnic, Carol went to the Deli and purchased: 0.52 pounds of salami at $3.29/pound; 0.48 pounds of sliced turkey at $8.99/pound; 1.04 pounds of ham at $3.99/pound; and 0.98 pounds of cheese at $4.29/pound. What was the total amount Carol spent at the deli? (Round your answer to the nearest cent.)

Teaching Notes:
- Most students have forgotten the mechanics of long division with decimals.
- Stress the importance of neatness so that the decimal ends up in the correct position. Some students find it helpful to do the division on graph paper or on lined paper that is rotated so the lines are vertical.
- Remind students when rounding the quotient to a specific place, you need to carry your division one more place than the rounding place.

Answers: 1a) 0.3, b) 0.301, c) -0.0018, d) 2.5, e) 39.1, f) 40; 2a)7.4; b)14.75;   3a) 0.774, b) -0.88773, c) 0.01047; 4a) -5.0$\overline{3}$, b) 0.5, c) -10.0$\overline{6}$; 5a) -21.81, b) $14.38

Mini-Lecture 5.5
Fractions, Decimals, and Order of Operations

Learning Objectives:

1. Write fractions as decimals.
2. Compare decimals and fractions.
3. Simplify expressions containing decimals and fractions using order of operations.
4. Solve area problems containing fractions and decimals.
5. Evaluate expressions given decimal replacement values.

Examples:

1. Write each fraction or mixed number as a decimal.

   a) $\dfrac{3}{5}$

   b) $-\dfrac{3}{20}$

   c) $\dfrac{1}{3}$

   d) $\dfrac{5}{16}$

   e) $-\dfrac{13}{11}$

   f) $-1\dfrac{7}{8}$

2. Insert <, >, or = to form a true statement.

   a) -0.0832 _____ -0.0823

   b) 0.501 _____ $\dfrac{1}{2}$

   c) 0.428 _____ $\dfrac{3}{7}$

   Write the numbers in order from smallest to largest.

   d) 0.331, $\dfrac{1}{3}$, 0.330

   e) 2.15, 2.142, $\dfrac{15}{7}$

   f) 1.5833, $1\dfrac{21}{36}$, $\dfrac{38}{36}$

3. Simplify each expression.

   a) $(0.3)^2 - 0.4$

   b) $(7.3)(100) - (7.2)(10)$

   c) $\dfrac{4 + 0.42}{-2}$

   d) $\dfrac{1}{4} - 3(6.5)$

   e) $\dfrac{3}{5} - (6.4)(-3)$

   f) $\dfrac{1}{5}(-9.1 - 3.3)$

4. a)
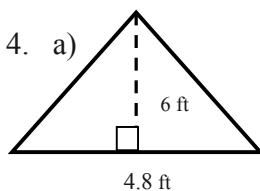
   5. Evaluate for $x = -2$, $y = 0.5$, and $z = 3.6$:

   a) $\dfrac{x}{y} + 2z$

Teaching Notes:

- Most students need a review of order of operations.
- Most students, once taught how to convert from fraction to decimal, have little problems.
- Some students have difficulty ordering numbers when they are mixed. Suggest that after converting to decimal, line the decimal points up vertically and compare corresponding place value.

Answers: 1a) 0.6, b) -0.15, c) $0.\overline{3}$, d) 0.3125, e) $-1.\overline{18}$, f) -1.875; 2a) <, b) > c) <, d) 0.330, 0.331, 1/3, e) 2.142, 15/7, 2.15, f) 38/36, 1.5833, 1 21/36; 3a) -0.31, b) 658, c) -2.21, d) -19.25, e) 19.8, f) -2.48; 4a)14.4ft; 5a) 3.2

Mini-Lecture 5.6
Solving Equations Containing Decimals

Learning Objectives:

    1. Solve equations containing decimals.

Examples:

    1. Solve each equation.

        a. $z + 0.8 = 2.5 =$

        b. $0.27x = -0.81 =$

        c. $3.9 = 1.5 + 0.6x$

        d. $5x + 3.6 = 8x + 12.9$

        e. $6.8 - 4x = 5(x - 13.7)$

Teaching Notes:
- Review steps for solving equations involving a variable.
- Stress neatness of lining up decimals when adding or subtracting, especially when an integer is present.
- Remind students the advantage of multiplying by powers of 10 to eliminate decimals when solving equations.

Answers: *1a) 1.7, b) -0.3, c) 4, d) -3.1, e) 3.7*

Mini-Lecture 5.7
Decimal Applications: Mean, Median, and Mode

Learning Objectives:

1. Find the mean of a list of numbers.
2. Find the median of a list of numbers.
3. Find the mode of a list of numbers.
4. Key Vocabulary: *measures of central tendency, mean, median, mode, grade point average (GPA), weighted mean.*

Examples:

1. Find the mean of the list of numbers. If necessary, round to the nearest tenth.

   a) 71, 47, 71, 99, 47   b) 143, 83, 225, 16   c) 4.1, 1.4, 8.7, 1.9, 13.1, 7.6

   d) Find the GPA (grade point average) for a particular student. Use the following for point values of the grades: A–4, B–3, C-2, D-1, F-0. If necessary, round the grade point average to the nearest hundredth.

Grade	Credit Hours
A	3
B	3
A	4
C	3

2. Find the median of the following list of numbers. Round to the nearest hundredth, if necessary.

   a) 3, 3, 14, 27, 31, 37, 50   b) 10, 2, 1, 28, 45, 48, 36

   c) 8, 2, 21, 19, 23, 49, 39, 38   d) 0.1, 3.3, 2.5, 0.3, 4, 2.7

3. Find the mode of the following list of numbers.

   a) 20, 43, 46, 43, 49, 43, 50   b) 90, 57, 32, 57, 29, 90

   c) 5, 9, 46, 3, 2, 8, 18, 1, 6, 19   d) 7.08, 7.41, 7.56, 7.08, 7.88, 7.99, 7.62

Teaching Notes:

- Most students are familiar with mean (average) but have not worked with median or mode.
- Encourage students to estimate when finding the mean.
- Stress the importance of ordering the information when finding median and mode.

Answers: *1a) 67, b) 116.8, c) 6.1, d) 3.31; 2a) 27, b) 28, c) 22, d) 2.6; 3a) 43, b) 57,90, c) no mode, d) 7.08*

Mini-Lecture 6.1
Ratio and Rates

Learning Objectives:

1. Write ratios as fractions.
2. Write rates as fractions.
3. Find unit rates.
4. Find unit prices.
5. Key Vocabulary: *ratio, rates, unit rate, miles per hour, unit price.*

Examples:

1. Write each ratio as a fraction in simplest form.

   a) 20 to 30

   b) 16 to 10

   c) 1.5 to 10

   d) 1.17 days to 9.9 days

   e) $3\frac{1}{4}$ hours to 6 hours

   f) $7\frac{1}{5}$ to $7\frac{1}{3}$

2. Write each rate as a fraction in simplest form.

   a) 81 miles in 42 minutes

   b) 1036 miles in 63 hours

   c) 6 tests for 30 students

   d) 246 miles on 54 gallons

3. Write each rate as a unit rate.

   a) 84 miles in 4 hours

   b) 468 miles on 18 gallons of gas

   c) 1161 cars in 387 households

   d) 380 people in 10 buses

4. Find each unit price. Round to the nearest cent, if necessary.

   a) $0.90 for 10 ounces

   b) $1.39 for 17 ounces

   c) $11.92 for 8 pounds of apples

   d) $118.58 for 121 pens

   e) Which is the better buy (lowest cost per ounce)? Round to the thousandth, if
      necessary. Shampoo: $0.70 for 11 ounces or 17 ounces for $1.39.

   f) There were 3 men and 6 women on a volleyball team. Find the ratio of men to
      women.

Teaching Notes:

- Some students need a quick review on converting mixed numbers to improper fractions and dividing fractions, decimals, etc.
- Remind students that order is important when solving ratio and rate word problems.
- Remind students that unit rate has a denominator of one.

Answers: 1a) 2/3, b) 8/5, c) 3/20, d) 13/110, e) 13/24, f) 54/55; 2a) 27miles/14minutes, b) 148 miles/9hours,
c) 1 test for 5 students, d) 41 miles per 9 gallons; 3a) 21 miles/hr, b) 26 miles/gallons, c) 3 cars/household,
d) 38 people/bus; 4a) $0.09/oz, b) $0.08/oz, c) $1.49/pound, d) $0.98/pen; e) 11 ounces for $0.07, f) 1/2

Mini-Lecture 6.2
Proportions

Learning Objectives:

1. Write sentences as proportions.
2. Determine whether proportions are true.
3. Find an unknown number in a proportion.
4. Key Vocabulary: *proportion, cross products.*

Examples:

1. Write each sentence as a proportion.

   a) 70 pencils is to 28 students as 5 pencils is to 2 students
   b) 96 guests is to 12 tables as 8 guests is to 1 table.
   c) $2\frac{1}{11}$ ounces of pasta is to 3 grams of protein as $12\frac{6}{11}$ ounces of pasta is to 18 grams of protein.

2. Determine whether each proportion is a true proportion.

   a) $\dfrac{3}{6} = \dfrac{4}{8}$
   b) $\dfrac{1\frac{10}{13}}{3} = \dfrac{3\frac{7}{13}}{6}$
   c) $\dfrac{0.6}{0.5} = \dfrac{3.0}{2.6}$
   d) $\dfrac{2\frac{2}{9}}{7} = \dfrac{8\frac{8}{9}}{30}$

3. For each proportion, find the unknown number n.

   a) $\dfrac{n}{54} = \dfrac{1}{18}$
   b) $\dfrac{1}{-2} = \dfrac{n}{15}$
   c) $\dfrac{1}{9\frac{1}{2}} = \dfrac{n}{19}$
   d) $\dfrac{-4.5}{n} = \dfrac{-2.5}{5.5}$

Teaching Notes:

- It is important to stress the easy problems 1a, 1b, and 2a to show that they are equivalent fractions.
- Many students have difficulty when complex fractions are involved in the proportion.
- Many students need to actually circle the cross products so they can be clear of the procedure.

Answers: 1a) $\dfrac{70}{28} = \dfrac{5}{2}$, *b)* $\dfrac{96}{12} = \dfrac{8}{1}$, *c)* $\dfrac{2\frac{1}{11}}{3} = \dfrac{12\frac{6}{11}}{18}$ *; 2a) yes, b) yes, c) no, d) no; 3a) 3, b) -7.5 or 7 1/2, c) 2, d) 9.9*

Mini-Lecture 6.3
Proportions and Problem Solving

Learning Objectives:

1. Solve problems by writing proportions.

Examples:

1. On a map of the city of Worcester, 1-inch corresponds to 12 miles. How far away is a town if the distance on the map measures 3 inches?

2. The ratio of students to faculty is 25 to 2. How many faculty members will be needed for a student population of 623? Round to the nearest whole number.

3. A particular piece of machinery made 1200 revolutions in 20 minutes. How many revolutions will occur in 23 minutes?

4. Four cups of water is needed to make 9.2 cups of rice. How many cups of rice can be made with 21 cups of water?

5. If 2 servings of a recipe calls for $\frac{3}{4}$ teaspoon of butter, how many servings can be made from 9 teaspoons of butter?

6. One bag of lawn & garden fertilizer covers 1000 square feet of lawn. How many bags of fertilizer must you purchase to cover a lawn 440 feet by 220 feet? Remember, you cannot purchase a fractional part of a bag.

Teaching Notes:

- Many students have trouble correctly setting up the proportion. Students tend to write the numbers as they appear rather than "lining up" the units.
- Remind students to estimate to see if their answer is reasonable.
- Refer students to the four steps for problem solving: Understand, Translate, Solve, and Interpret.

Answers: 1) 36 miles, 2) 50 faculty members; 3) 1380 revolutions; 4) 48.3 cups; 5) 24 servings; 6) 97 bags.

Mini-Lecture 6.4
Square Roots and the Pythagorean Theorem

Learning Objectives:

1. Find the square root of a number.
2. Approximate square roots.
3. Use the Pythagorean Theorem.
4. Key Vocabulary: *square root, radical sign, positive square root, Pythagorean Theorem, leg, hypotenuse.*

Examples:

1. Find each square root.

   a) $\sqrt{4}$ b) $\sqrt{25}$ c) $\sqrt{49}$ d) $\sqrt{81}$

   e) $\sqrt{\dfrac{1}{64}}$ f) $\sqrt{\dfrac{16}{25}}$ g) $\sqrt{\dfrac{25}{49}}$

2. Use a calculator or the appropriate Appendix to approximate each square root to the thousandths position.

   a) $\sqrt{2}$ b) $\sqrt{6}$ c) $\sqrt{11}$ d) $\sqrt{39}$

3. Sketch each right triangle and find the length of the side not given. If necessary, round the answer to the nearest thousandth.

   a) leg = 3, leg = 4   b) leg = 12, hypotenuse = 15   c) hypotenuse = 6.4, leg = 3

   Solve. If necessary, round to the nearest thousandth.

   d) A section of land is a square with each side measuring 2 miles. Find the length of the diagonal of the section of land.

   e) A garden is in the shape of a rectangle. The diagonal length of the garden is 25 feet, and the length of one of the sides is 15 feet. Find the length of the other side.

Teaching Notes:

- Some students have never done square roots on a calculator and will need guidance.
- When approximating square roots, encourage students to mentally estimate the answer. That way if they use the calculator incorrectly they might be able to notice the incorrect result.
- Most students do not have trouble using the Pythagorean Theorem for finding a hypotenuse, but some have trouble using it for finding a missing leg.

Answers: 1a) 2, b) 5, c) 7, d) 9, e) 1/8, f) 4/5, g) 5/7; 2a) 1.414, b) 2.449, c) 3.317, d) 6.245; 3a) 5, b) 9, c) 5.653; d) 2.828 mi, e) 20 ft

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Mini-Lecture 6.5
Congruent and Similar Triangles

Learning Objectives:

1. Decide whether two triangles are congruent.
2. Find the ratio of corresponding sides in similar triangles.
3. Find unknown lengths of sides in similar triangles.
4. Key Vocabulary: *congruent, Angle-Side-Angle (ASA), Side-Side-Side (SSS), Side-Angle-Side (SAS), similar.*

Examples:

1. Determine whether each pair of triangles is congruent.
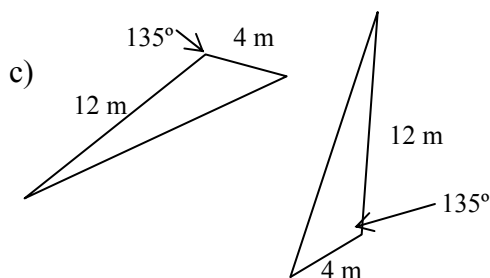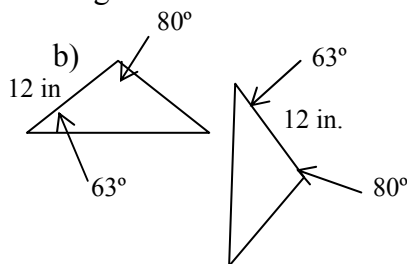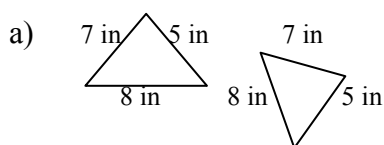
2. Find each ratio of the corresponding sides of the given similar triangles.
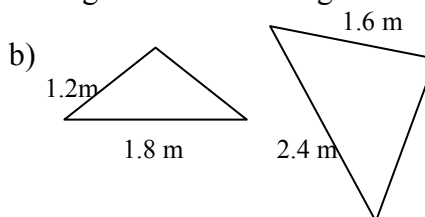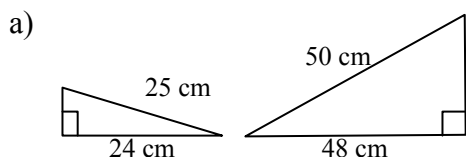
3. Solve. Round to the nearest tenth.

   a) A tree casts a shadow of 26 ft. Nearby, a 5-ft pole casts a shadow of 3 ft. What is the height of the tree?
   b) A rock climber is 6 feet tall and his shadow measures 9 feet long. The rock he wants to climb casts a shadow of 580 feet. How tall is the rock?

Teaching Notes:

- Some students need a review on solving proportions.
- Encourage students to mentally visualize applied problems and draw and label a diagram before solving.

Answers: 1a) yes, b) yes, c) yes; 2a) ½, b) 4/3; 3a) 43 1/3 ft., b) 386 2/3 ft

Mini-Lecture 7.1
Percents, Decimals, and Fractions

Learning Objectives:
1. Understand percent.
2. Write percents as decimals or fractions.
3. Write decimals or fractions as percents.
4. Solve applications with percents, decimals, and fractions.
5. Key Vocabulary: *percent.*

Examples:
1. Write the percent described by the sentence.
   In a recent survey of 100 students, 37 of the students indicated that they eat lunch at the dining hall. What percent of the students surveyed do not eat lunch in the dining hall?

2. Write each percent as a decimal.
   a) 32%   b) 5.7%   c) 120%   d) 0.02%

   Write each percent as a fraction or mixed number in simplest form.

   e) 15%   f) 8.1%   g) 275%   h) $5\frac{1}{2}\%$

3. Write each decimal as a percent.
   a) 0.77   b) 2.6   c) 0.081   d) 3.00

   Write each fraction or mixed number as a percent.

   e) $\frac{23}{50}$ f) $\frac{3}{5}$ g) $\frac{7}{12}$ h) $4\frac{2}{5}$

Complete the table. Round to the nearest thousandth, if necessary

	Percent	Fraction	Decimal
i)		$\frac{3}{20}$	
j)	27%		
k)			0.55
l)		$2\frac{1}{6}$	

4. Write each percent as a decimal and a fraction.
   a) A family decides to spend no more than 27.5% of its monthly income on rent. Write 27.5% as a decimal and a fraction.

   b) Provincetown's budget for waste disposal increased by $1\frac{3}{4}$ times over the budget from last year. What percent increase is this?

Teaching Notes:
- Some students have trouble remembering which way to move the decimal point.
- Some students become very confused with converting between percents, fractions, and decimals, and need to see many examples.

Answers: 1) 37%, 63%; 2a) 0.32, b) 0.057, c) 1.2, d) 0.0002, e) 3/20, f) 81/1000, g) 2 ¾ , h) 11/200; 3a) 77%, b) 260%,

c) 8.1%, d) 300%, e) 46%, f) 60%, g) $58.\overline{3}\%$ or $58\frac{1}{3}\%$, h) 440%, i) 15%, 0.15, j) 27/100, 0.27, k) 55%, 11/20,

l) 216.7%, 2.167; 4a) 0.275,11/40, b)175%

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Mini-Lecture 7.2
Solving Percent Problems with Equations

Learning Objectives:

1. Write percent problems as equations.
2. Solve percent problems.
3. Key Vocabulary: *of, is, what number, percent equation..*

Examples:

1. Translate each to an equation. Do not solve.

   a) 12% of 80 is what number?

   b) What percent of 60 is 20?

   c) 40% of what number is 20?

   d) 1.8 is what percent of 9?

2. Solve the following equations for the amount.

   a) What number is 8% of 50?

   b) 20% of 65 is what number?

   c) 125% of 16 is what number?

   d) What number is $15\frac{3}{4}$% of 50?

   e) 70% of what number is 35?

   f) 18 is 4% of what number?

   g) 8.1 is 36% of what number?

   h) 22.5% of what number is 2.7?

   i) What percent of 200 is 16?

   j) 0.06 is what percent of 100?

   k) 210 is what percent of 60?

   l) What percent of 1041 is 333.12?

Teaching Notes:

- Many students get confused between "amount" and "base".
- Some students have better success at solving these types of equations by working in Section 6.4, Solving Percent Problems Using Proportions.

Answers: 1a) (0.12)(80) =x, b) x• 60=20, c) 0.40 x = 20, d) 1.8 = 9x; 2a) 4, b) 13, c) 20, d) 7.875; e) 50, f)450, g) 22.5, h) 12; i) 8%, j) 0.06%, k) 350%, l) 32%

Mini-Lecture 7.3
Solving Percent Problems with Proportions

Learning Objectives:

1. Write percent problems as proportions.
2. Solve percent problems.
3. Key Vocabulary: *percent (p), base (b), amount(a), percent proportion* $\left(\dfrac{a}{b} = \dfrac{p}{100}\right)$.

Examples:

1. Translate each to a proportion. Use *n* to indicate the unknown. Do not solve.

   a) 50% of 24 is what number?   b) What percent of 40 is 14?

   c) 60% of what number is 2?   d) What percent of 50 is 50?

2. Solve.

   a) 30% of 350 is what number?   b) What number is 8% of 625?

   c) What number is 250% of 60?   d) 20% of 8.5 is what number?

   e) 8% of what number is 4?   f) 84 is 70% of what number?

   g) $5\dfrac{1}{2}$ % of what number is $2\dfrac{1}{5}$?   h) 1716 is 143% of what number?

   i) What percent of 120 is 90?   j) 616 is what percent of 560?

   k) 2.1 is what percent of 70?   l) What percent of 800 is 6?

Teaching Notes:

- Remind students that the percent proportion is: $\dfrac{amount}{base} = \dfrac{percent}{100}$
- Many students find it difficult to identify the different parts of the proportion.
- Remind students that the "base" is after the word "of".

Answers: 1a) $\dfrac{50}{100} = \dfrac{n}{24}$ *, b)* $\dfrac{14}{40} = \dfrac{n}{100}$ *, c)* $\dfrac{2}{n} = \dfrac{60}{100}$ *, d)* $\dfrac{50}{50} = \dfrac{n}{100}$ *; 2a) 105, b) 50, c) 150, d) 1.7; e) 50, f) 120, g) 40,*
h) 1200 i) 75%, j) 110%, k) 3%, l) 0.75%

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Mini-Lecture 7.4
Applications of Percent

Learning Objectives:

1. Solve applications involving percent.
2. Find the percent increase and percent decrease.
3. Key Vocabulary: *percent increase, percent decrease.*

Examples:

1. Solve.
   a) 15% of Carol's check of $1200 is paid towards health care. How much money is paid towards health care?

   b) During a recent inspection, the fire department found 112 faulty smoke alarms. If this is 0.08% of the total inspected, how many smoke alarms were inspected?

   c) One day, 18 students were out sick with the flu. What percent of the students were absent if there should be a total of 80 students in the class?

2. Solve.

   a) Find the percent of increase if the original amount was 80 and the new amount is 100.

   b) Find the percent of increase if the original amount was 20 and the new amount is 65.

   c) Recently, a bookstore announced that all their books would increase in price 5%. How much will a book cost if the original price was $4.50. Round to the nearest cent.

   d) Find the percent of decrease if the original amount was 16 and the new amount is 10.

   e) Find the percent of decrease if the original amount was 140 and the new amount is 91.

   f) On a recent shopping trip, a sign read: "Jeans! Originally $40, Now $34!". Find the percent decrease in price.

Teaching Notes:

- Most students find this section difficult. Refer students to the following formulas:

$$\% \, increase = \frac{amount \ of \ increase}{original \ amount} \quad \text{or} \quad \% \, decrease = \frac{amount \ of \ decrease}{original \ amount}$$

Answers: 1a) $180, b) 140,000, c) 22.5%; 2a) 25%, b) 225%, c) $4.73; d) 37.5%, e) 35%, f) 15%

Mini-Lecture 7.5
Percent and Problem Solving: Sales Tax, Commission, and Discount

Learning Objectives:

1. Calculate sales tax and total price.
2. Calculate commissions.
3. Calculate discount and sale price.
4. Key Vocabulary: *sales tax, sales tax rate, total price, commission, commission rate, amount of discount, sale price.*

Examples:

1. Solve.
    a) Find the sales tax and the total price on the purchase of a $230 DVD player where the sales tax rate is 6.5%.

    b) The sales tax on a $1050 computer system is $63. What is the sales tax rate?

    c) The portable DVD player costs $185 and the leather carrying case costs $30. What is the total paid to the cashier if the sales tax rate is 7%?

2. Solve.
    a) A book salesman is paid a commission of 3.1% of her monthly sales. For the month of August, she sold $180,000 worth of books. What was the amount of her commission for the month?

    b) A salesperson earned a commission of $3,842.50 for selling $53,000 worth of merchandise. Find the salesperson's commission rate.

3. Solve.
    a) Find the amount of discount when the original price is $72 and the discount rate is 20%.

    b) Find the sale price when the original price is $58 and the discount rate is 9%.

    c) A $3300 diamond bracelet is part of a "25% off" sale. Find the discount and the sale price for the bracelet.

Teaching Notes:

*   Most students find these types of problems difficult and frustrating.
*   Refer students to the following formulas:
    Sales Tax = tax rate · purchase price
    Total Price = purchase price + sales tax
    Commission = commission rate · sales
    Amount of Discount = discount rate · original price
    Sale Price = original price – amount of discount

Answers: 1a) $14.95, $244.95, b) 6%, c) $230.05; 2a) $5580, b) 7.25% or 7 ¼%; 3a) $14.40, b) $52.78, c) $825, $2475

Mini-Lecture 7.6
Percent and Problem Solving: Interest

Learning Objectives:

1. Calculate simple interest.
2. Calculate compound interest.
3. Key Vocabulary: *simple interest; principal; rate; time; total amount of a loan; compound interest; compounded- annually, semi-annually, quarterly; compound interest factor.*

Examples:

1. Solve.

   a) Upon graduation, Ayla is given money that totals $4,700. If this money is invested at 9.5% simple interest for 7 years, find the total amount.

   b) Allison borrows $10,500 for $4\frac{1}{2}$ years at a rate of 9.5% simple interest. Find the total amount.

   c) $250,000 is borrowed to buy a house. If the simple interest rate on the 30-year loan is 6.75%, find the total amount paid on the loan.

2. Solve. Round answers to the nearest cent.

   a) Kristen deposited $2,850 in a compound interest account for 5 years. If the account earns 10% interest compounded quarterly, find the total amount. (Compound interest factor is 1.63862)

   b) $4,690 is compounded semi-annually at a rate of 8%. Find the total amount of compound interest earned at the end of 15 years. (Compound interest factor is 3.2434).

   c) $2500 is compounded daily at a rate of 8% for 10 years. Find the total amount.

Teaching Notes:

- Refer students to the following formulas in the textbook:
  Simple Interest = principal · rate · time
  Total Amount = principal + interest
  Compound Interest Tot. Amt. =original principal · compound interest factor
  Monthly Payment = $\dfrac{\text{principal} + \text{interst}}{\text{total number of payments}}$
- Refer students to Appendix A7 for the table of compound interest factors.

Answers: 1a) $7825.50, b) $14,988.75, c) $756,250; 2a) $4670.07, b) $15,211.54, c) $5563.36.

Mini-Lecture 8.1
Reading Pictographs, Bar Graphs, Histograms and Line Graphs

Learning Objectives:
1. Read pictographs.
2. Read and construct bar graphs.
3. Read and construct histograms.
4. Read line graphs.
5. Key Vocabulary: *pictograph, bar graph, histogram, class interval, class frequency, line graph.*

Examples:

1. The following bar graph shows the points scored per quarter in a basketball game.
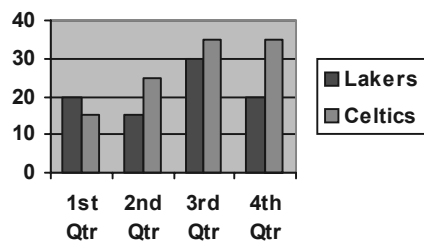
a) How many points did the Celtics score in the 4th quarter?
b) How many points did the Lakers score in the 1st quarter?
c) What was the total score for the game?

3. The following line graph shows the total sales per month.
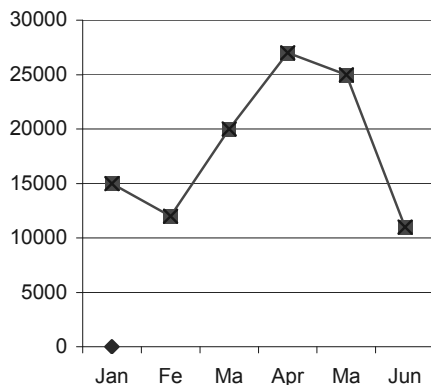
a) What month had the least sales?
b) What is the difference between the highest and lowest month?

2. The following pictograph shows the number of satisfied customers at a restaurant. Use the information to answer the following questions.

Month	Number of Satisfied Customers
Jan.	☺☺☺☺☺☺
Feb.	☺☺☺
Mar.	☺☺☺☺

☺ = 100 Satisfied Customers

a) Which month had the most satisfied customers?
b) What was the total number of satisfied customers during the 3-month period?

4. The following is a list of scores on a recent math exam. Use this list to complete the frequency distribution table: 100, 85, 89, 75, 60, 55, 92, 85, 85, 76, and 77

	Class Intervals	Tally	Class Frequency
a	90 - 100		
b	80 - 89		
c	70 - 79		
d	60 - 69		
e	Below 60		

Teaching Notes:
- Encourage students to look around their environment and find different graphs.

Answers: 1a) 35, b) 20, c) L=85, C=110; 2a) Jan, b) 1300;3a) June, b) 16,500; 4a) 2,4,3,1,1

Mini-Lecture 8.2
Reading Circle Graphs

Learning Objectives:

1. Read circle graphs.
2. Draw circle graphs.
3. Key Vocabulary: *circle graph, protractor, 360°.*

Examples:

1. The freshman class held elections for class president. The following circle graph shows the results of how the 200 students voted. Use the circle graph to answer the accompanying questions.

Votes For Class President

Ray 26%
Amy 33%
Haiwin 15%
Chris 26%
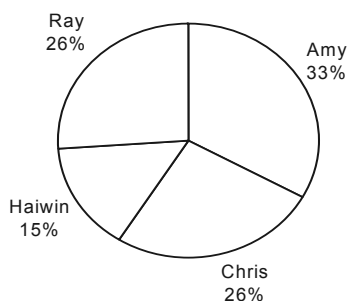
a) Who won the election?

b) Who received the fewest votes?

c) How many votes did the winner receive?

d) Write a ratio of students voting for Haiwin to total number of students.

2. A recent survey of college students asked how many cans of soda did they consume per day. Complete the table and draw a circle graph to represent the information given in the table.

	Number of Cans of Soda Consumed in One Day	Percent of College Students	Degrees of a Circle
a	0	5%	
b	1 - 2	42%	
c	3 - 4	20%	
d	5 - 6	15%	
e	7 or more	18%	

Teaching Notes:

- Remind students that the order of a ratio is important.
- Some students will need instruction on the use of a protractor.
- Encourage students, when drawing a circle graph, to estimate first.

Answers: *1a) Amy, b) Haiwin, c) 66, d) 3/20; 2a) 18°, b) 151.2°, c) 72°, d) 54°, e) 64.8°*

Mini-Lecture 8.3
The Rectangular Coordinate System and Paired Data

Learning Objectives:

1. Plot points on a rectangular coordinate system.
2. Determine whether ordered pairs are solutions of equations.
3. Complete ordered pair solutions of equations.
4. Key Vocabulary *ordered pair, origin, quadrant, x-axis, y-axis, rectangular coordinate system, coordinate plane, x-coordinate, y-coordinate, paired data, solution of an equation in two variables.*

Examples:

1. Plot each order pair. State in which quadrant or on which axis each point lies.

   a) (-2, -5)   b) (0, -4)   c) $\left(2\frac{2}{3}, 4\frac{1}{2}\right)$ d) (-1, 4)

2. Determine whether each ordered pair is a solution of the given linear equation.

   a) $x - y = 4$; (1, 2)   b) $x + 3y = 7$; (4,1)   c) $y = 5x + 7$; (-2, -3)

3. Complete each ordered pair so that it is a solution of the given linear equation.

   a) $x + 2y = 6$; (2,   ), (, -3)   b) $y = \frac{1}{3}x - 2$; (6,   ), $\left(, -\frac{1}{3}\right)$

Teaching Notes:
- Many students have trouble putting meaning to an ordered pair.
- Remind students that an ordered pair is (x, y) – alphabetical order.

Answers: 1a) III, b) y-axis, c) I, d) II; 2a) no, b) yes, c) yes; 3a) (2, 2), (12, -3), b) (6, 0), (5, $-\frac{1}{3}$)

Mini-Lecture 8.4
Graphing Linear Equations in Two Variables

Learning Objectives:

1. Graph linear equations by plotting points.
2. Key Vocabulary: *linear equation in two variables, graph of the equation, horizontal line, vertical line.*

Examples:

1. For each equation, find three ordered pair solutions by completing the table. Then use the ordered pairs to graph the equation.

a) $x - y = 2$

x	y
3	
	-2
-1	

b) $y = -\dfrac{1}{3}x - 2$

x	y
6	
	-4
	0

c) $y = \dfrac{2}{3}x$

x	y
-6	
	0
	$\dfrac{10}{3}$

d) $y = -3$

x	y
2	
-1	
0	

Graph the following linear equations.

e) $x + y = 0$　　　　f) $y = -2x - 1$　　　　g) $x - 2 = 0$

Teaching Notes:

- Problems 1a) – d) tend to pose the least amount of challenge.
- Some students become very confused when they can choose any value for x or y as a starting point for finding an ordered-pair solution.
- Many students do not understand problems 1d) and 1g) and must memorize the form for an equation for a horizontal or a vertical line.

Answers (for all graphs, see Mini-Lecture graphing answers at end of section) : 1a) (3,1), (0, -2), (-1, -3);

1b) (6, -4), (6, -4), (-6, 0); 1c) (-6, -4), (0,0), (5, $\dfrac{10}{3}$); 1d) (-2,), (-3, -1), (0, -3); 1e) – 1g) see graphing answers

Mini-Lecture 8.5
Counting and Introduction to Probability

Learning Objectives:

1. Use a tree diagram to count outcomes.
2. Find the probability of an event.
3. Key Vocabulary: *probability, experiment, outcomes, tree diagram, event, probability of an event.*

Examples:

1. Draw a tree diagram to find the number of possible outcomes.

   a) Choose a number: 2, 4, or 6, and then a letter: a, b, or c.

   b) Choose a shirt: red or blue, and then pants: white, black, or brown.

   c) Toss a coin and then toss a single six-sided dice.

   d) Toss four coins.

2. Find the probability of each event occurring.

   a) If a single die is tossed, find the probability of a 2.

   b) If a single die is tossed, find the probability of a number greater than 2.

   c) If a single choice is made from a bag of marbles containing 2 red, 2 blue, 2 green, find the probability of choosing a red marble; find the probability of choosing a red or green marble.

   d) Recently, a weight loss drug was tested on 300 people. The results were as follows: 220 people lost weight; 60 people stayed the same weight, 20 people gained weight. What is the probability that a person taking this drug would lose weight? What is the probability that a person would not lose weight (either stay the same or gain)?

Teaching Notes:

- Refer students to the textbook for sample tree diagrams and the formula for **The Probability of an Event.**
- Most students, once shown tree diagrams, are able to satisfactorily draw a tree diagram.
- Some students need to list or draw the possible outcomes to find or visualize the probability.

Answers: 1a) 9, b) 6, c) 12, d) 16; 2a) 1/6, b) 2/3, c) 1/3, 2/3, d) 11/15, 4/15

Mini-Lecture 9.1
Lines and Angles

Learning Objectives:

1. Identify lines, line segments, rays, and angles.
2. Classify angles as acute, right, obtuse, or straight.
3. Identify complementary and supplementary angles.
4. Find measures of angles.
5. Key Vocabulary: *space, plane, point, line, line segment, ray, angle, vertex, sides, degrees, straight angle, right angle, acute angle, obtuse angle, complementary angles, supplementary angles, parallel lines, intersecting lines, perpendicular, vertical angles, adjacent angles, transversal, corresponding angles are equal, alternate interior angles are equal.*

Examples:

1. Draw an example of each term.

   a) line   b) ray   c) segment   d) angle

2. Draw an example of each angle.

   a) acute   b) right   c) obtuse   d) straight

3. Find each complementary or supplementary angle as indicated.

   a) Find the complement of a 35º angle.   b) Find the complement of a 71º angle.
   c) Find the supplement of a 152º angle.   d) Find the supplement of a 83º angle.

4. Find the measures of angles x, y, and z in each figure.

   a)   122º   b) $m \parallel n$ 139º
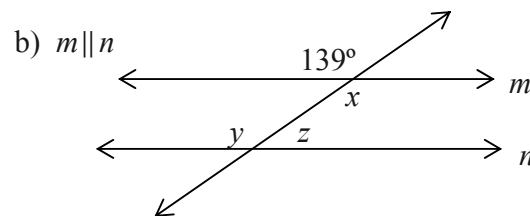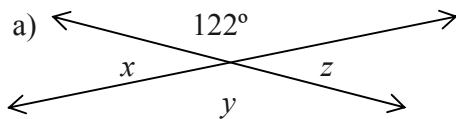
Teaching Notes:

- Some students are unfamiliar with the vocabulary and need repetition.
- Refer students to the textbook, **Chapter Highlights**, for a condensed listing of definitions and concepts.

Answers: 1a) – d) Answers may vary; 2a – d) Answers may vary; 3a) 55°, b) 19°c) 28°, d) 97°; 4a) x =58°, y=122°, z=58°, b) x=139°, y=139°, z=41°

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Mini-Lecture 9.2
Perimeter

Learning Objectives:

1. Use formulas to find perimeters.
2. Use formulas to find circumferences.
3. Key Vocabulary: *perimeter, circumference, π*.

Examples:

1. Find the perimeter of the following figures.

   a)   15 m   b)   25 ft   c)   10 ft

   6 m   23 ft   23 ft   8 ft   7 ft

                              18 ft   2 ft
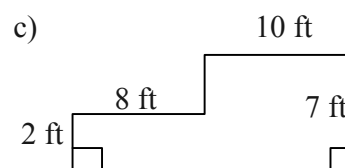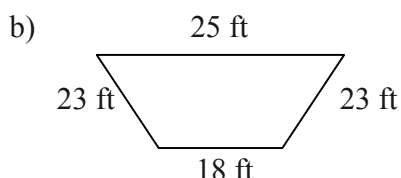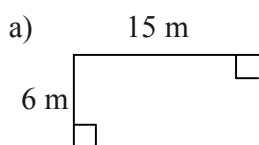
   d) A rectangular field measures 441 feet by 108 feet. Find the cost of constructing
      a fence if fencing costs $29.50 per yd.

2. Find the circumference of each circle. Give the exact and then an approximation by using
   $\pi = 3.14$. Round to the nearest hundredth.

   a) radius = 7 ft.   b) diameter = 84 m.   c) radius = 34.9 ft

   d) A circular room has a radius of 10.3 feet. Find the distance around the room.

   e) A circular statue has a base with a diameter of 11 ft. Find the distance around the base
      of the statue.

   f) Find the distance around a circular Jacuzzi with a diameter of 8 ft. For this problem,
      use $\pi = \dfrac{22}{7}$

Teaching Notes:

- Refer students to textbook for formulas of perimeter and circumference.
- Many students have difficulty understanding the difference between approximation and exact
  value when working with π.
- Remind students to read carefully and take note of radius vs. diameter.

Answers: 1a) 42 m, b) 89 ft., c) 50 ft., d) $10,797; 2a) 14$\pi$, 43.96 ft, b) 84$\pi$, 263.76 m, c) 69.8$\pi$, 219.17 ft,
d) 20.6π, 64.69 ft, e) 11π, 34.54 ft., f) 8π, 27 1/7 ft.

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Mini-Lecture 9.3
Area, Volume, and Surface Area

Learning Objectives:
1. Find the area of plane regions.
2. Find the volume and surface area of solids.
3. Key Vocabulary: *area, square units*.

Examples:
1. Find the area of the following. Use $\pi = 3.14$. Round to the nearest hundredth.

   a) 7 yd, 3 yd (rectangle) b) 8 ft, 3.8 ft (triangle) c) d=24 miles (circle)
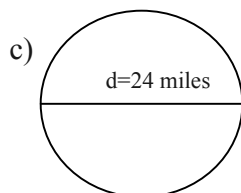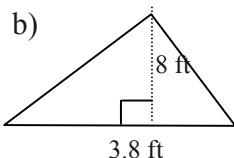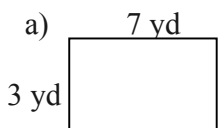
   e) A small rug is in the shape of a trapezoid. The bases measure 19.8 in and 22.4 in. and the height is 8 in. Find the area of the rug.

   f) A side of a square towel measures 7.5 in. Find how many square inches of material is needed to make 8 towels.

2. Find the volume and surface area of each rectangular solid or cube. Round to the nearest tenth.
   a) Rectangular solid: length = 8.3 in.; width = 3 in.; height = 9 in.
   b) Cube: side = 7.5 cm

   Find the volume and surface area of each sphere or circular cylinder. Give the exact answers and an approximate. Use $\pi = 3.14$. Round to the nearest hundredth.
   c) Sphere: diameter = 12 in.
   d) Circular cylinder: radius = 8 feet; height = 3 feet

   Find the volume and surface area of each cone or square-based pyramid. If necessary, give the exact answer and an approximate. Use $\pi = 3.14$. Round to the nearest hundredth.
   e) Cone: height = 4 m; radius = 3 m
   f) Square-based pyramid: height = 11 cm; edge of base = 10.5 cm; slant height = 12.19 cm

Teaching Notes:
- Refer students to textbook for **Area Formulas of Common Geometric Figures** and **Area Formula of a Circle.**
- Many students have difficulty when the height lies outside of a triangle, parallelogram, or trapezoid.
- Refer students to the textbook for **Volume and Surface Area Formulas of Common Solids**.
- Some students need to be able to visualize the solids. Encourage students to seek out actual items in their environment for these common solids.
- Remind students that volume is measured in cubic units, and surface area in square units.

Answers: 1a) 21 yd ^{2}, b) 15.2 ft^2, c) 452.16 mi^2, d) 109.88 cm^2, e) 168.8 in^2, f) 450 in^2, : 2a) V=224.1 in^3, SA=253.2 in^2, b) V=421.9 cm^3, SA=337.5 cm^2; c) V=288 π in^3 ≈ 904.32 in^3, SA=144 π in^2 ≈ 452.16 in^2, d) V=192π ft^3 ≈ 602.88 ft^3, SA=176 π ft^2 ≈ 552.64 ft^2; e) V= 12π m^3 ≈ 37.70 m^3, SA=24 π m^2 ≈ 75.40 m^2,f) V=404.25 cm^3, SA=366.24 cm^2

Mini-Lecture 9.4
Linear Measurement

Learning Objectives:

1. Define U.S. units of length and convert from one unit to another.
2. Use mixed U.S. units of length.
3. Perform arithmetic operations on U.S. units of length.
4. Define metric units of length and convert from one unit to another.
5. Perform arithmetic operations on metric units of length.
6. Key Vocabulary: *U.S. or English measurement system, metric system, inch, foot, yard, mile, unit fraction, kilo, hector, deka, meter, deci, centi, milli.*

Examples:

1. Convert each measurement as indicated.
   a) 72 in to feet   b) 8 yd to feet   c) 3 mi to feet   d) 120 in to yd

2. Convert each measurement as indicated.

   a) 89 in = __ ft __ in   b) 4 yd 2 ft = __ in   c) 11,213 ft = ___ mi ___ ft

3. Perform each indicated operation. Simplify the result if possible.

   a) 11 ft 3 in. + 5 ft 10 in.   b) 42 yd 1 ft – 38 yd 2 ft   c) 18 ft 8 in. ÷ 2

   d) A garden is 7 ft. 11 in. long by 8 ft. 7 in. wide. What is the total length of fencing is needed to completely enclose the garden?

4. Convert as indicated.

   a) 60 mm to cm   b) 500 m to km   c) 8.2 m to cm

   d) 34,000 mm to m   e) 3.7 km to m   f) 50.6 mm to dm

5. Perform each indicated operation. Simplify the result if possible.

   a) 26 cm + 11.9m   b) 33 mm – 1.443 cm   c) 4.3 mm ÷ 5

   d) A 2.8-m board has 1.3 cm trimmed from each end. How long is the remaining board?

Teaching Notes:

- Refer students to **U.S. Units of Length** chart and **Metric Units of Length** chart.
- Remind students to always put the unit they are converting to in the numerator of the unit fraction.

Answers: 1a) 6 ft., b) 24 ft., c) 15,840 ft., d) 3 1/3 yd.; 2a) 7 ft. 5 in, b) 168 in., c) 2 mi 653 ft; 3a) 17 ft 1 in, b) 3 yd 2 ft, c) 9 ft 4 in, d) 33 ft; 4a) 6 cm, b) 0.5 km, c) 820 cm, d) 34 m, e) 3700 m, f) 0.506 dm; 5a) 12.16 m, b) 18.57 mm, c) 0.86 mm, d) 277.4 cm or 2.774 m

Mini-Lecture 9.5
Weight and Mass

Learning Objectives:

1. Define U.S. units of weight and convert from one unit to another.
2. Perform arithmetic operations on U.S. units of weight.
3. Define metric units of mass and convert from one unit to another.
4. Perform arithmetic operations on units of mass.
5. Key Vocabulary: *weight, ounce, pound, ton, mass, gram, milligram, kilogram.*

Examples:

1. Convert as indicated.

   a) 192 oz = _?_ lb

   b) 8,800 lbs = _?_ ton

   c) $\frac{2}{5}$ oz = _?_ lb

   d) 6.7 lb = _?_ oz

   e) 16 lb 3 oz = _?_ oz

   f) 3.1 ton = _?_ oz

2. Perform each indicated operation.

   a) 37 lb 12 oz + 22 lb 7 oz

   b) 59 lb 2 oz − 18 lb 15 oz

   c) 16 tons 1400 lb ÷ 5

   d) A company wishes to ship 15 boxes of books. If each box weighs 3 lb 10 oz, what is the total weight of 15 boxes?

3. Convert as indicated.

   a) 27 kg = _?_ g

   b) 310 g = _?_ kg

   c) 22 g = _?_ mg

   d) 1,035 mg = _?_ g

   e) 8,360 cg = _?_ kg

   f) 16 hg = _?_ mg

4. Perform each indicated operation.

   a) 11.7 mg + 3.2 mg

   b) 5 g − 1301 mg

   c) 9 kg ÷ 4

   d) A bottle weighs 125 grams. Find the weight in kilograms of 2 dozen bottles.

Teaching Notes:

- Refer students to the **U.S. Units of Weight** chart and the **Metric Units of Mass** chart in the textbook.
- Review the use of unit fractions.
- Remind students that the prefixes are the same in the metric system for mass and length.

Answers: 1a) 12 lb, b) 4.4 ton, c) 1/40 or 0.025 lb, d) 107.2 oz, e) 259 oz, f) 99.2 oz; 2a) 60 lb 3 oz, b) 40 lb 3 oz, c) 6680 lb, d) 54 lb 6 oz; 3a) 27,000g, b) 0.31 kg, c) 22,000 mg, d) 1.035 g, e) 0.0836 kg, f) 1,600,000 mg.;
4a) 14.9 mg, b) 3.699g, c) 2.25 kg, d) 3 kg

Mini-Lecture 9.6
Capacity

Learning Objectives:

1. Define U.S. units of capacity and convert from one unit to another.
2. Perform arithmetic operations on U.S. units of capacity.
3. Define metric units of capacity and convert from one unit to another.
4. Perform arithmetic operations on metric units of capacity.
5. Key Vocabulary: *capacity, fluid ounces, cup, pint, quart, gallon, liter, kilo, hector, deka, deci, centi, milli.*

Examples:

1. Convert each measurement as indicated.

   a) 72 fl oz = _?_ c   b) 10 pt = _?_ c   c) $6\frac{1}{2}$ qt = _?_ pt

   d) 26 qt = _?_ gal   e) $3\frac{1}{8}$ qt = _?_ fl oz   f) 5 gal = _?_ fl oz

2. Perform each indicated operation.

   a) 8 gal 5 qt + 6 gal 2 qt   b) 5 pt – 2 pt 1 c   c) 3 gal 8 cups × 3

   d) A recipe calls for 45 fluid ounces of water. How many cups is this?

3. Convert each measurement as indicated.

   a) 6 L = _?_ ml   b) 3.2 L = _?_ cl   c) 1800 ml = _?_ L

   d) 0.135 L = _?_ kl   e) 0.072 dl = _?_ ml   f) 43,000 L = _?_ hl

4. Perform each indicated operation.

   a) 17.5 L + 16.8 L   b) 12,520 ml – 0.6 L   c) 13.5 L ÷ 1.8

   d) A chemistry student accidentally added 69 cl of a chemical to a mixture instead of 38 ml. How much extra of the chemical did the student add?

Teaching Notes:

- Refer students to **U.S. Units of Capacity chart** and **Metric Unit of Capacity chart** in textbook.
- Remind students to use unit fractions whenever possible.
- With metrics, many students will memorize prefixes and move the decimal point accordingly.

Answers: 1a) 9c, b) 20c, c) 13 pt, d) 6.5 gal, e) 100 fl oz, f) 640 fl oz; 2a) 15 gal 3 qt, b) 2 pt 1 c, d) 10 gal 8 c; 3a) 6000 ml, b) 320 cl, c) 1.8 L, d) 0.000135 kl, e) 7.2 ml, f) 430 hl; 4a) 34.3 L, b) 11.92L, c) 7.5L, d) 652 ml

Mini-Lecture 9.7
Temperature and Conversions Between the U.S. and Metric Systems

Learning Objectives:

1. Convert between the U.S. and metric systems.
2. Convert temperatures from degrees Celsius to degrees Fahrenheit.
3. Convert temperatures from degrees Fahrenheit to degrees Celsius.

Examples:

1. Convert as indicated. If necessary, round answers to two decimal places.

   a) 12 km = _?_ mi   b) 6 in = _?_ cm   c) 5.5 ft = _?_ m

   d) 20 L = _?_ gal   e) 34 qt = _?_ L   f) 27 L = _?_ qt

   g) 3 kg = _?_ lb   h) 16 oz = _?_ g   i) 14.3 lb = _?_ kg

2. Covert from degrees Celsius to degrees Fahrenheit. When necessary, round to the nearest tenth of a degree.
   a) 60°C to degrees Fahrenheit
   b) 32°C to degrees Fahrenheit
   c) 50°C to degrees Fahrenheit
3. Convert from degrees Fahrenheit to degrees Celsius. When necessary, round to the nearest Tenth of a degree.
   a) 68° F to degrees Celsius
   b) 113° F to degrees Celsius
   c) 20° F to degrees Celsius

Teaching Notes:

- Remind students to use their unit fractions.
- Refer students to the textbook for conversions involving length, capacity, and weight.
- Some students may need to find objects that correspond to the conversions so they can visualize these equivalences.

Answers: 1a) 7.44 mi., b) 15.24 cm., c) 1.65 m., d) 5.2 gal, e) 32.3 L, f) 28.62 qt, g) 6.6 lb., h) 453.6 g, i) 6.44 k;
2a) 140° F, b) 89.6°F, c) 122° F; 3a)20° C b)45°C, c)-6.7° C

Mini-Lecture 10.1
Adding and Subtracting Polynomials

<u>Learning Objectives:</u>
1. Add polynomials.
2. Subtract polynomials.
3. Evaluate polynomials at given replacement values.

<u>Examples:</u>
1. Add the polynomials.
   a) $(3y + 7) + (-9y - 14)$
   b) $(x^2 - 4x - 3) + (5x^2 - 6x)$
   c) $(-z^2 - 4.2z + 11) \, and \, (9z^2 - 1.9z + 6.3)$

2. Subtract the polynomials.
   a) $(9b + 8) - (11b - 20)$
   b) $(-4a + 20) - (-14a + 36)$
   c) $(11x^2 + 7x + 2) - (15x^2 + 4x)$
   d) Subtract $(-7y^2 + y - 4) \, from \, (-3y^2 + 5y)$
   e) Subtract $(3x^2 - 12x) \, from \, (-4x^2 + 20x + 17)$

3. Find the value of each polynomial when y = 3.
   a) $2y^3 + y^2 - 6$
   b) $\dfrac{4y^2}{2} - 14$
   c) $4y^2 - 5y + 10$

<u>Teaching Notes:</u>
- Stress the definition of like terms.
- Stress the concept of distributing the negative in front of the parentheses.
- Stress turn around words, such as the word *from*.

Answers: 1a) -6y – 7, b) 6x² – 10x – 3, c) 8z² – 6.1z + 17.3; 2a) -2b + 28, b) 10a - 16, c) -4x² + 3x + 2 ,
d) 4y² + 4y + 4, e)-7x² + 32x + 17; 3a) 57, b) 4, c) 31

Mini-Lecture 10.2
Multiplication Properties of Exponents

Learning Objectives:

1. Use the product property for exponents.
2. Use the power property for exponents.
3. Use the power of a product property for exponents.
4. Key Vocabulary: *exponential expression, power, raised, product rule, same base, simplifying an exponential expression, power rules, quotient rule.*

Examples:

1. Use the product property to simplify each expression. Write the results using exponents.

   a) $x^5 \cdot x^3$ b) $(4z^3)(9z^5)$ c) $(-3x^3y^2)(-5x^4y^6)$ d) $(9ab^2c^4)(-11a^3b)(-2b^2c^5)$

2. Use the power property to simplify each expression.

   a) $(x^7)^3$ b) $(y^3)^{11}$ c) $(xy)^5$ d) $(m^7)^5 \cdot (m^2)^3$

3. Use the power of a product property to simplify each expression.

   a) $(-3x)^3 (2x^3)^4$ b) $(-3x)(2x^2)^4$ c) $(2x)(-2x^3y^2)^2$ d) $(2x^2y^4)^4(3x^6y^9)^2$

Teaching Notes:
- Most students need a lot of practice to master these properties.

Answers 1a) x^8; *b)* $36z^8$; *c)* $15x^7y^8$; *d)* $198a^4b^5c^9$; *2a)* x^{21}; *b)* y^{33}; *c)* x^5y^5; *d)* m^{41}; *3a)* $-432x^{15}$; *b)* $-48x^9$; *c)* $8x^7y^4$; *d)* $144x^{20}y^{34}$

Mini–Lecture 10.3
Multiplying Polynomials

<u>Learning Objectives:</u>
1. Multiply a monomial and any polynomial.
2. Multiply two binomials.
3. Square a binomial.
4. Use the FOIL order to multiply binomials.
5. Multiply any two polynomials.

<u>Examples:</u>

4. Multiply.

   a) $4y(8y^2 + 5)$ 　　　　b) $-2a(2a^2 - a + 12)$ 　　　　c) $3r(8r^2 - r + 11)$

5. Multiply.

   a) $(x+4)(x+5)$ 　　　　b) $(b+3)(b+5)$ 　　　　c) $(7x-1)(5x+4)$

6. Multiply.

   a) $(b+3)^2$ 　　　　b) $(y-3)^2$ 　　　　c) $(6y-1)^2$

7. Use FOIL order to multiply.

   a) $(10x-7)(2x+3)$ 　　　　b) $(3x+2)^2$

8. Multiply.

   a) $(x+3)(x^2 + 2x - 2)$ 　　　　b) $(2x+5)(x^2 + 4x - 1)$

<u>Teaching Notes:</u>
- Remind students: FOIL order can only be used to multiply two binomials.
- Show several examples on squaring a binomial to reinforce using the definition of an exponent.

Answers: 1a) $32y^3 + 20y$ *, b)* $-4a^3 + 2a^2 - 24a$ *, c)* $24r^3 - 3r^2 + 33r$ *; 2a)* $x^2 + 9x + 20$ *, b)* $b^2 + 8b + 15$ *,*
c) $35x^2 + 23x - 4$ *; 3a)* $b^2 + 6b + 9$ *, b)* $y^2 - 6y + 9$ *, c)* $36y^2 - 12y + 1$ *; 4a)* $20x^2 + 16x - 21$ *, b)*
$9x^2 + 12x + 4$ *; 5a)* $x^3 + 5x^2 + 4x - 6$ *, b)* $2x^3 + 13x^2 + 18x - 5$

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Mini-Lecture 10.4
Introduction to Factoring Polynomials

Learning Objectives:

1. Find the greatest common factor of a list of integers.
2. Find the greatest common factor of a list of terms.
3. Factor the greatest common factor from the terms of a polynomial.
4. Key Vocabulary: *factors, factored form, factoring, factoring out, greatest common factor, GCF, factoring by grouping.*

Examples:

1. Find the greatest common factor for each list.

   a) 16, 6   b) 18, 24   c) 15, 21   d) 12, 28, 40

2. Find the GCF for each list.

   a) $15m^2, 25m^5$ b) $40x^2, 20x^7$ c) $-28x^4, 56x^5$ d) $21m^2n^5, 35mn^4$

3. Factor out the GCF from each polynomial.

   a) $5a + 15$ b) $56z + 8$ c) $y^3 + 2y$

   d) $5x^3 + 10x^4$ e) $16z^5 + 8z^3 - 12z$ f) $x(y^2 - 2) + 3(y^2 - 2)$

   g) $6a^8b^9 - 8a^3b^4 + 2a^2b^3 + 4a^5b^3$

Teaching Notes:

- Many students remove common factors, not the *greatest* common factor.
- Encourage students to factor in a step-by-step manner: first factor out the GCF for the coefficients, then the GCF for each variable.
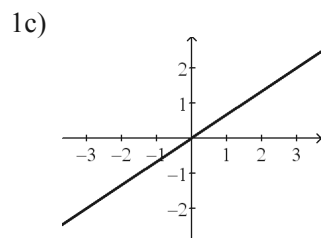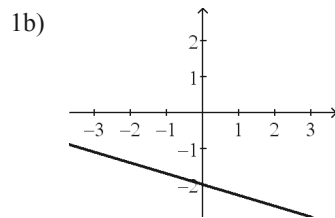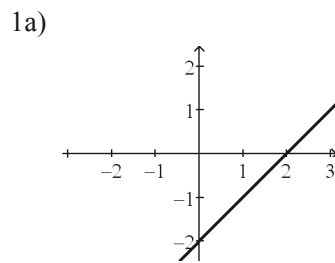- Remind students that they can check their work by multiplying.

Answers: *1a) 2; b) 6; c) 3; d) 4; 2a) $5m^2$; b) $20x^2$; c) $28x^4$; d) $7mn^4$; 3a) 5(a+3); b) 8(7z+1); c) $y(y^2+2)$; d) $5x^3(1+2x)$; e) $4z(4z^4+2z^2-3)$; 3f) $(y^2-2)(x+3)$; g)$2a^2b^3(3a^6b^6-4ab+1+2a^3$*

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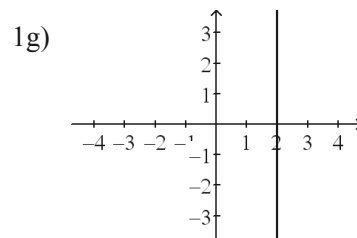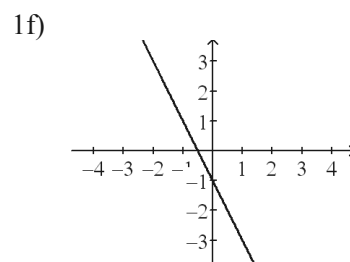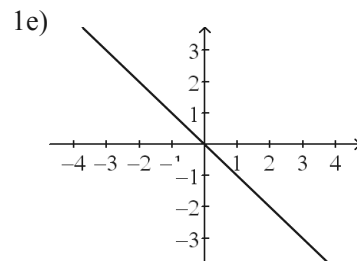
Mini-Lecture Graphing
Answers

Section 8.4 continued

1d)

Section 4.1

3a)

3b)

3c)

3d)

1e)

Section 8.4

1a)

1f)

1b)

1g)

1c)
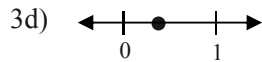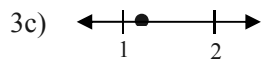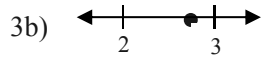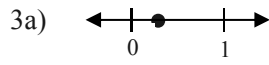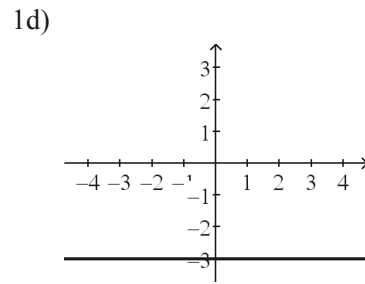
2a) *x* axis: 1 mark = 1 year
   y axis: 1 mark = $10,000