

Test #3: sections 5.7 through 7.7

The following exam is closed notes, closed book, and most importantly closed neighbor. Work quickly so that you can go back and check your work.

1. Solve. $\left(\frac{x^2-23}{2x^2-5x-3} + \frac{2}{x-3} = \frac{-1}{2x+1}\right) (2x+1)(x-3)$

$$x^2 - 23 + 4x + 2 = -x + 3 \quad \left| \quad \begin{array}{l} x + 8 = 0 \quad \vee \quad x - 3 = 0 \\ \underline{-8} \quad \underline{-8} \quad \quad \underline{+3} \quad \underline{+3} \end{array} \right.$$

$$x^2 + 5x - 24 = 0 \Rightarrow \boxed{x = -8} \quad \cancel{x = 3}$$

$$(x+8)(x-3) = 0$$

2. Divide. $(x^4 + 4x^3 - x^2 - 16x - 4) \div (x - 2)$

$$\begin{array}{r} 2 \overline{) 1 \ 4 \ -1 \ -16 \ -4} \\ \underline{1 \ 2} \\ 2 \\ \underline{2 \ 12} \\ 22 \\ \underline{22} \\ 0 \end{array} = \boxed{x^3 + 6x^2 + 11x + 6 + \frac{8}{x-2}}$$

3. Add. $\frac{2x}{3x^2-13x+4} + \frac{5}{x^2-2x-8} = \frac{(2x)(x+2)}{(3x-1)(x-4)(x+2)} + \frac{5(3x-1)}{(x-4)(x+2)(3x-1)}$

$$= \frac{2x^2 + 4x}{(3x-1)(x-4)(x+2)} + \frac{15x - 5}{(3x-1)(x-4)(x+2)}$$

$$= \boxed{\frac{2x^2 + 19x - 5}{(3x-1)(x-4)(x+2)}}$$

4. Divide. $\frac{2a^2-2b^2}{a^3+a^2b+a+b} \div \frac{6a^2}{a^3+a} = \frac{2(a^2-b^2)}{(a^3+a^2b)+(a+b)} \cdot \frac{a(a^2+1)}{6a^2}$

$$= \frac{2(a-b)(a+b)}{a^2(a+b)+1(a+b)} \cdot \frac{a(a^2+1)}{6a^2} = \frac{2(a-b)\cancel{(a+b)} a(a^2+1)}{\cancel{(a+b)}(a^2+1) \cancel{6a^2}}$$

$$= \boxed{\frac{a-b}{3a}}$$

5. Factor completely. $250y^3 - 16x^3$

$$= 2(125y^3 - 8x^3)$$

$$= \boxed{2(5y - 2x)(25y^2 + 10xy + 4x^2)}$$

6. Divide.

$$\frac{(7+3i)(2-4i)}{(2+4i)(2-4i)} = \frac{14-28i+6i-12i^2}{4-16i^2} = \frac{14-22i+12}{4+16} = \frac{26-22i}{20} = \frac{2(13-11i)}{20} = \frac{13-11i}{10} = \frac{13}{10} - \frac{11i}{10}$$

7. Solve

$$2x + \sqrt{x+1} = 8$$

$$\begin{aligned} (\sqrt{x+1})^2 &= (8-2x)^2 \\ x+1 &= 64 - 32x + 4x^2 \\ 4x^2 - 33x + 63 &= 0 \end{aligned}$$

$$(4x-21)(x-3) = 0$$

$$4x-21=0 \vee x-3=0$$

$$4x=21$$

$$x = \frac{21}{4}$$

$$\text{check } x = \frac{21}{4} \\ 2\left(\frac{21}{4}\right) + \sqrt{\frac{21}{4}+1} \stackrel{?}{=} 8 \\ \frac{21}{2} + \frac{5}{2} \neq 8$$

$$x = 3$$

check $x=3$

$$2(3) + \sqrt{3+1} \stackrel{?}{=} 8$$

$$6 + \sqrt{4} \stackrel{?}{=} 8$$

$$6 + 2 = 8 \checkmark$$

8. Rationalize the denominator.

$$\frac{\sqrt{a+1}}{2\sqrt{a}-\sqrt{b}}$$

$$\frac{(\sqrt{a+1})(2\sqrt{a}+\sqrt{b})}{(2\sqrt{a}-\sqrt{b})(2\sqrt{a}+\sqrt{b})}$$

$$= \frac{2a + \sqrt{ab} + 2\sqrt{a} + \sqrt{b}}{4a - b}$$

9. Add.

$$\sqrt{4x^7y^5} + 9x^2\sqrt{x^3y^5} + 5xy\sqrt{x^5y^3}$$

$$= \sqrt{4x^6y^4 \cdot xy} + 9x^2\sqrt{x^2y^4 \cdot xy} + 5xy\sqrt{x^4y^2 \cdot xy}$$

$$= 2x^3y^2\sqrt{xy} + 9x^3y^2\sqrt{xy} + 5x^3y^2\sqrt{xy}$$

$$= 16x^3y^2\sqrt{xy}$$

10. Simplify.

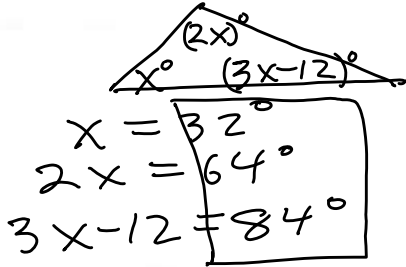
$$\sqrt[3]{40x^{10}}$$

$$= \sqrt[3]{8x^9 \cdot 5x} = 2x^3\sqrt[3]{5x}$$

11. Write as a single radical expression.

$$= y^{\frac{1}{6}} y^{\frac{1}{3}} y^{\frac{2}{5}} = y^{\frac{1}{6} + \frac{1}{3} + \frac{2}{5}} = y^{\frac{5}{30} + \frac{10}{30} + \frac{12}{30}} = y^{\frac{27}{30}} = y^{\frac{9}{10}} = \sqrt[10]{y^9}$$

12. Find the measures of the angles of a triangle if the measure of one angle is twice the measure of a second angle and the third angle measures 3 times the second angle decreased by 12.



$$x + (2x) + (3x - 12) = 180$$

$$6x - 12 = 180$$

$$\begin{array}{r} 6x - 12 = 180 \\ +12 \quad +12 \\ \hline 6x = 192 \\ \hline x = 32 \end{array}$$

13. Bob takes 12 hours to do a task. Working with his son they can complete the task in 7 hours. How long will it take his son working alone to complete the task?

$$84x \left(\frac{1}{12} + \frac{1}{x} = \frac{1}{7} \right)$$

$$7x + 84 = 12x$$

$$\begin{array}{r} 7x + 84 = 12x \\ -7x \quad -7x \\ \hline 84 = 5x \\ \hline x = 16.8 \end{array}$$

$$x = 16 \text{ hrs} + (.8) \text{ hr} (60 \text{ min/hr})$$

$$x = 16 \text{ hrs} + 48 \text{ min}$$

16:48

14. Bob, the chemist, needs 600 gallons of 30% alcohol solution. He only has supplies of 20% alcohol solution and 50% alcohol solution to draw from. How much of each should he use in his mixture?

$$\boxed{20\%} + \boxed{50\%} = \boxed{30\%}$$

$$x \text{ gal} \quad (600-x) \text{ gal} \quad 600 \text{ gal}$$

$$.2x + .5(600-x) = .3(600)$$

$$.2x + 300 - .5x = 180$$

$$\begin{array}{r} .2x + 300 - .5x = 180 \\ -300 \quad -300 \\ \hline -.3x = -120 \\ \hline x = 400 \end{array}$$

$$\begin{array}{r} -.3x = -120 \\ \hline x = 400 \end{array}$$

400 gal of 20%
200 gal of 50%

15. The speed of a boat in still water is 24 mph. If the boat travels 54 miles upstream in the same time that it takes to travel 90 miles downstream, find the speed of the current.

$$d = rt \Rightarrow t = \frac{d}{r}$$

$$\frac{54}{24-c} = \frac{90}{24+c}$$

$$54(24+c) = 90(24-c)$$

$$1296 + 54c = 2160 - 90c$$

$$\begin{array}{r} 1296 + 54c = 2160 - 90c \\ -1296 + 90c \quad -1296 + 90c \\ \hline 144c = 864 \\ \hline c = 6 \end{array}$$

let $c =$ current in mph

$$c = 6$$

6 mph