**Intermediate Algebra**
**Skill-Builder # AE - 9**
**Expanding Binomials Using Pascal's Triangle**

$$(a+b)^0:\qquad\qquad 1$$
$$(a+b)^1:\qquad\qquad 1\quad 1$$
$$(a+b)^2:\qquad\qquad 1\quad 2\quad 1$$
$$(a+b)^3:\qquad 1\quad 3\quad 3\quad 1$$
$$(a+b)^4:\qquad 1\quad 4\quad 6\quad 4\quad 1$$
$$(a+b)^5:\quad 1\quad 5\quad 10\quad 10\quad 5\quad 1$$
$$(a+b)^6:\ 1\quad 6\quad 15\quad 20\quad 15\quad 6\quad 1$$
$$(a+b)^7:1\quad 7\quad 21\quad 35\quad 35\quad 21\quad 7\quad 1$$

<u>Examples</u> Expand using Pascal's Triangle.

**1.** $(x+2)^5$

Solution:

The expansion will have **6 terms** with **coefficients 1, 5, 10, 10, 5, 1**. The terms will be in **descending powers of $x$** starting from $x^5$ and ending in $x^0$ or 1 and **ascending powers of 2** starting from $2^0$ or 1 and ending in $2^5$. Thus,

$$(x+2)^5 = 1\cdot x^5\cdot 2^0 + 5\cdot x^4\cdot 2^1 + 10\cdot x^3\cdot 2^2 + 10\cdot x^2\cdot 2^3 + 5\cdot x^1\cdot 2^4 + 1\cdot x^0\cdot 2^5$$

and simplifying we get

$$(x+2)^5 = x^5 + 10x^4 + 40x^3 + 80x^2 + 80x + 32.$$

**2.** $(3x-2y)^6$

Solution:

The expansion will have **7 terms** with **coefficients 1, 6, 15, 20, 15, 6, 1**. The terms will be in descending powers of $3x$ starting from $(3x)^6$ and ending in $(3x)^0$ or 1 and ascending powers of $-2y$ starting from $(-2y)^0$ or 1 and ending in $(-2y)^6$. Thus,

$$(3x-2y)^6 = 1\cdot(3x)^6(-2y)^0 + 6\cdot(3x)^5(-2y)^1 + 15\cdot(3x)^4(-2y)^2 + 20\cdot(3x)^3(-2y)^3 + 15\cdot(3x)^2(-2y)^4$$
$$+ 6\cdot(3x)^1(-2y)^5 + 1\cdot(3x)^0(-2y)^6$$

Simplifying, we get

$$(3x-2y)^6 = 729x^6 - 2916x^5y + 4860x^4y^2 - 4320x^3y^3 + 2160x^2y^4 - 576xy^5 + 64y^6$$

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Expand using Pascal's Triangle.

**1.** $(x+y)^3$

**2.** $(x-4)^3$

**3.** $(y+4)^4$

**4.** $(n-2)^4$

**5.** $(a+3)^5$

**6.** $(x-2)^6$

**7.** $(3a+2b)^5$

**8.** $(2c-3d)^6$

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**Answers**

1. $x^3 + 3x^2y + 3xy^2 + y^3$

2. $x^3 - 12x^2 + 48x - 64$

3. $y^4 + 16y^3 + 96y^2 + 256y + 256$

4. $n^4 - 8n^3 + 24n^2 - 32n + 16$

5. $a^5 + 15a^4 + 90a^3 + 270a^2 + 405a + 243$

6. $x^6 - 12x^5 + 60x^4 - 160x^3 + 240x^2 - 192x + 64$

7. $243a^5 + 810a^4b + 1080a^3b^2 + 720a^2b^3 + 240ab^4 + 32b^5$

8. $64c^6 - 576c^5d + 2160c^4d^2 - 4320c^3d^3 + 4860c^2d^4 - 2916cd^5 + 729d^6$