Calculus Formulas

Power Rules: $\frac{d}{dx}x^n = nx^{n-1}$ and $\int x^n dx = \frac{x^{n+1}}{n+1} + c$	**Product Rule:** $\frac{d}{dx}[f(x)\cdot g(x)] = f(x)\cdot g'(x) + f'(x)\cdot g(x)$
Quotient Rule: $\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{g(x)\cdot f'(x) - f(x)\cdot g'(x)}{[g(x)]^2}$	**Reciprocal Rule:** $\frac{d}{dx}\left[\frac{1}{g(x)}\right] = \frac{-g'(x)}{[g(x)]^2}$
Chain Rule: $\frac{d}{dx}(f \circ g)(x) = f'[g(x)]\cdot g'(x)$	**Integration-by-Parts:** $\int u\,dv = uv - \int v\,du$

Trigonometric Functions

Derivative	Integral
$\frac{d}{dx}\sin x = \cos x$	$\int \sin x\,dx = -\cos x + c$
$\frac{d}{dx}\cos x = -\sin x$	$\int \cos x\,dx = \sin x + c$
$\frac{d}{dx}\tan x = \sec^2 x$	$\int \tan x\,dx = \ln\lvert\sec x\rvert + c$ $\int \sec^2 x\,dx = \tan x + c$
$\frac{d}{dx}\cot x = -\csc^2 x$	$\int \cot x\,dx = \ln\lvert\sin x\rvert + c$ $\int \csc^2 x\,dx = -\cot x + c$
$\frac{d}{dx}\sec x = \sec x\cdot\tan x$	$\int \sec x\,dx = \ln\lvert\sec x + \tan x\rvert + c$ $\int \sec x\cdot\tan x\,dx = \sec x + c$
$\frac{d}{dx}\csc x = -\csc x\cdot\cot x$	$\int \csc x\,dx = \ln\lvert\csc x - \cot x\rvert + c$ $\int \csc x\cdot\cot x\,dx = -\csc x + c$

Inverse Trigonometric Functions

Derivative	Integral
$\frac{d}{dx}\sin^{-1} x = \frac{1}{\sqrt{1-x^2}}$	$\int \frac{1}{\sqrt{a^2-u^2}}\,dx = \sin^{-1}\frac{u}{a} + c$
$\frac{d}{dx}\cos^{-1} x = \frac{-1}{\sqrt{1-x^2}}$	
$\frac{d}{dx}\tan^{-1} x = \frac{1}{1+x^2}$	$\int \frac{1}{a^2+u^2}\,dx = \frac{1}{a}\tan^{-1}\frac{u}{a} + c$
$\frac{d}{dx}\cot^{-1} x = \frac{-1}{1+x^2}$	
$\frac{d}{dx}\sec^{-1} x = \frac{1}{\lvert x\rvert\sqrt{x^2-1}}$	$\int \frac{1}{u\sqrt{u^2-a^2}}\,dx = \frac{1}{a}\sec^{-1}\frac{u}{a} + c$
$\frac{d}{dx}\csc^{-1} x = \frac{-1}{\lvert x\rvert\sqrt{x^2-1}}$	

Identities:

$$\sin^2 x + \cos^2 x = 1 \qquad \sin 2x = 2\sin x\cos x \qquad \cos^2 x = \frac{1+\cos 2x}{2}$$

$$1+\cot^2 x = \csc^2 x \qquad \cos 2x = \cos^2 x - \sin^2 x \qquad \sin^2 x = \frac{1-\cos 2x}{2}$$

$$\tan^2 x + 1 = \sec^2 x \qquad \cos(x+y) = \cos x\cos y - \sin x\sin y \qquad \sin(x+y) = \sin x\cos y + \cos x\sin y$$

Exponential Functions

Derivative	Integral
$\frac{d}{dx}(e^x) = e^x$	$\int e^x\,dx = e^x + c$
$\frac{d}{dx}(b^x) = (\ln b)b^x$	$\int b^x\,dx = \frac{b^x}{\ln b} + c$

Logarithmic Functions

Derivative	Integral
$\frac{d}{dx}(\ln\lvert x\rvert) = \frac{1}{x}$	$\int \frac{1}{x}\,dx = \ln\lvert x\rvert + c$
$\frac{d}{dx}(\log_b\lvert x\rvert) = \frac{1}{(\ln b)x}$	

Definition of Log base b: $\log_b N = x \Leftrightarrow b^x = N$

Change of Base Formula: $\log_b x = \frac{\ln x}{\ln b} = \frac{\log x}{\log b}$

Identities:

$$\ln(e^x) = x \qquad e^{\ln x} = x \qquad \ln e = \log 10 = \log_b b = 1$$

$$\log_b(b^x) = x \qquad b^{\log_b x} = x \qquad \ln 1 = \log 1 = \log_b 1 = 0$$

Infinite Series: Definitions & Tests

1. *Series*: $\begin{cases} \sum_{n=1}^{\infty} a_n = a_1 + a_2 + a_3 + \dots \ (\textbf{\textit{Infinite Series}}) \\ s_n = \sum_{i=1}^{n} a_i = a_1 + a_2 + \dots + a_n \ (\textbf{\textit{nth Partial Sum}}) \\ \textbf{\textit{if}} \lim_{n\to\infty} s_n = s \ \textbf{\textit{where}}\ s \in \Re \ \textbf{\textit{then}} \sum_{n=1}^{\infty} a_n = s \ (\textbf{\textit{Infinite Sum}}) \end{cases}$

2. *Geometric Series*: $\sum_{n=0}^{\infty} ar^n = a + ar + ar^2 + ar^3 + \dots = \begin{cases} \frac{a}{1-r}, \textbf{\textit{if}} \ |r| < 1 \\ \textit{diverges}, \textbf{\textit{if}} \ |r| \ge 1 \end{cases}$

3. *P-Series*: $\sum_{n=1}^{\infty} \frac{1}{n^p} \Rightarrow \begin{cases} \textit{converges}, \textbf{\textit{if}} \ p > 1 \\ \textit{diverges}, \textbf{\textit{if}} \ p \le 1 \end{cases}$ ***if p = 1, the series is called the harmonic series.***

4. *Quick Divergence Test*: Given $\sum_{n=1}^{\infty} a_n \Rightarrow \begin{cases} \textbf{\textit{if}} \lim_{n\to\infty} a_n \ne 0, \textbf{\textit{then}} \sum_{n=1}^{\infty} a_n \ \textbf{\textit{diverges}} \\ \textbf{\textit{if}} \lim_{n\to\infty} a_n = 0, \textbf{\textit{then No Conclusion! Do another test!}} \end{cases}$

5. *Integral Test:* Given $\sum_{n=c}^{\infty} a_n,\ a_n > 0, a_n$ ***decreasing*** $\Rightarrow \begin{cases} \textbf{\textit{if}} \int_c^{\infty} a_n \, dn \ \textbf{\textit{converges then}} \sum_{n=c}^{\infty} a_n \ \textbf{\textit{converges}} \\ \textbf{\textit{if}} \int_c^{\infty} a_n \, dn \ \textbf{\textit{diverges then}} \sum_{n=c}^{\infty} a_n \ \textbf{\textit{diverges}} \end{cases}$

6. *Ratio Test:* Given $\sum_{n=c}^{\infty} a_n,\ a_n > 0 \Rightarrow$ ***if*** $\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = p$ ***then*** $\begin{cases} \sum_{n=c}^{\infty} a_n \ \textbf{\textit{converges, when}} \ p < 1, \\ \sum_{n=c}^{\infty} a_n \ \textbf{\textit{diverges, when}} \ p > 1, \\ \textbf{\textit{No Conclusion, when}} \ p = 1 \end{cases}$

7. *Root Test*: Given $\sum_{n=c}^{\infty} a_n,\ a_n > 0 \Rightarrow$ ***if*** $\lim_{n\to\infty} \sqrt[n]{a_n} = \lim_{n\to\infty} (a_n)^{\frac{1}{n}} = p$ ***then*** $\begin{cases} \sum_{n=c}^{\infty} a_n \ \textbf{\textit{converges, when}} \ p < 1, \\ \sum_{n=c}^{\infty} a_n \ \textbf{\textit{diverges, when}} \ p > 1, \\ \textbf{\textit{No Conclusion, when}} \ p = 1 \end{cases}$

8. *Limit Comparison Test:* $\sum_{n=c}^{\infty} a_n$ ***and*** $\sum_{n=c}^{\infty} b_n,\ a_n > 0, b_n > 0 \Rightarrow \begin{cases} \textbf{\textit{if}} \lim_{n\to\infty} \frac{a_n}{b_n} = p,\ p > 0,\ p \ \textbf{\textit{finite}} \\ \textbf{\textit{then both series converge or both diverge}} \end{cases}$

9. *Comparison Test:* $\sum_{n=c}^{\infty} a_n$ ***and*** $\sum_{n=c}^{\infty} b_n,\ a_n \ge 0, b_n \ge 0, a_n \le b_n \Rightarrow \begin{cases} \textbf{\textit{if}} \ b_n \ \textbf{\textit{converges then}} \ a_n \ \textbf{\textit{converges,}} \\ \textbf{\textit{if}} \ a_n \ \textbf{\textit{diverges then}} \ b_n \ \textbf{\textit{diverges}} \end{cases}$

10. *Alternating Series Test*: Given $\sum_{n=c}^{\infty} (-1)^n a_n$, ***if*** $a_n > 0, a_{n+1} < a_n, \lim_{n\to\infty} a_n = 0,$ ***then*** $\sum_{n=c}^{\infty} (-1)^n a_n$ ***converges***